INSTITUTE OF TEXTILE TECHNOLOGY

FLUID MECHANICS

4th Semester

Branch: Mechanical Engg.

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1.1. INTRODUCTION

Fluid mechanics is that branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion. Thus this branch of science deals with the static, kinematics and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

1.2. PROPERTIES OF FLUIDS

1.2.1. Density or Mass Density. Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.* kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}.$$

The value of density for water is 1 gm/cm^3 or 1000 kg/m^3.

1.2.2. Specific Weight or Weight Density. Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol *w*.

Thus mathematically,

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$= \rho \times g \qquad \left\{ \because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho \right\}$$

$$\therefore \quad w = \rho g \qquad ...(1.1)$$

The value of specific weight or weight density (w) for water is 9.81×1000 Newton/m^3 in SI units.

1.2.3. Specific Volume. Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of a fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg. It is commonly applied to gases.

1.2.4. Specific Gravity. Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol S.

Mathematically, $$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus weight density of a liquid $= S \times$ Weight density of water

$= S \times 1000 \times 9.81 \text{ N/m}^3$

The density of a liquid $= S \times$ Density of water

$= S \times 1000 \text{ kg/m}^3$. ...(1.1 A)

Problem 1.1. *Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.*

Sol. Given :

Volume $= 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$

Weight $= 7$ N.

(*i*) Specific weight (w) $= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \text{ N/m}^3}$. **Ans.**

(*ii*) Density (ρ) $= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \text{ kg/m}^3}$. **Ans.**

(*iii*) Specific gravity $= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \ \text{Density of water} = 1000 \text{ kg/m}^3\}$

$= \mathbf{0.7135}$. **Ans.**

Problem 1.2. *Calculate the density, specific weight and weight of one litre of petrol of specific gravity = 0.7.*

Sol. Given :

Volume $= 1$ litre

$= 1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity, $S = 0.7$

(*i*) *Density* (ρ)

Using equation (1.1 A),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = \mathbf{700 \text{ kg/m}^3}$. **Ans.**

(*ii*) *Specific weight* (w)

Using equation (1.1), $w = \rho \times g$

$= 700 \times 9.81 \text{ N/m}^3 = \mathbf{6867 \text{ N/m}^3}$. **Ans.**

(*iii*) *Weight* (W)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $$w = \frac{W}{0.001} \quad \text{or} \quad 6867 = \frac{W}{0.001}$$

$\therefore$ $$W = 6867 \times 0.001 = \mathbf{6.867\ N.} \quad \textbf{Ans.}$$

1.3. VISCOSITY

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. When two layers of a fluid, a distance '*dy*' apart, move one over the other at different velocities, say *u* and *u* + *du* as shown in Fig. 1.1, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

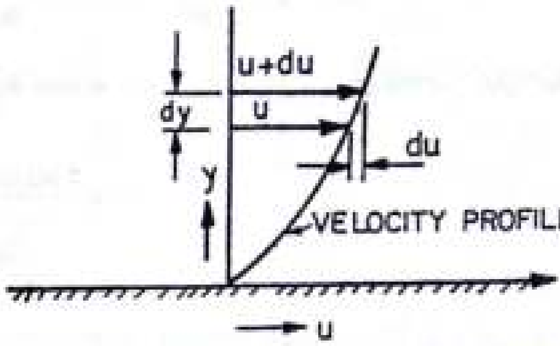


Fig. 1.1. Velocity variation near a solid boundary.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to *y*. It is denoted by symbol τ called Tau.

Mathematically, $$\tau \propto \frac{du}{dy}$$

or $$\tau = \mu \frac{du}{dy} \qquad \text{...(1.2)}$$

where μ (called *mu*) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.

From equation (1.2), we have $$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} \qquad \text{...(1.3)}$$

Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

1.3.1. Units of Viscosity. The units of viscosity is obtained by putting the dimensions of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\frac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\frac{\text{Length}}{\text{Time}}\right) \times \frac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(length)}^2}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}.$$

In MKS system, force is represented by kgf and length by metre (m), in CGS system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by metre (m).

$\therefore$ MKS unit of viscosity $= \frac{\text{kgf-sec}}{\text{m}^2}$

CGS unit of viscosity $= \frac{\text{dyne-sec}}{\text{cm}^2}$

SI unit of viscosity $= \dfrac{\text{Newton-sec}}{\text{m}^2} = \dfrac{\text{N s}}{\text{m}^2}$

The unit of viscosity in CGS is also called Poise which is equal to $\dfrac{\text{dyne-sec}}{\text{cm}^2}$.

The numerical conversion of the unit of viscosity from MKS unit to CGS unit is given below :

$$\frac{\text{one kgf-sec}}{\text{m}^2} = \frac{9.81 \text{ N-sec}}{\text{m}^2} \qquad \{\because \ 1 \text{ kgf} = 9.81 \text{ Newton}\}$$

But one Newton $= \text{one kg (mass)} \times \text{one} \left(\dfrac{\text{m}}{\text{sec}^2}\right) \text{(acceleration)}$

$$= \frac{(1000 \text{ gm}) \times (100 \text{ cm})}{\text{sec}^2} = 1000 \times 100 \frac{\text{gm-cm}}{\text{sec}^2}$$

$$= 1000 \times 100 \text{ dyne} \qquad \left\{\because \ \text{dyne} = \text{gm} \times \frac{\text{cm}}{\text{sec}^2}\right\}$$

$$\therefore \quad \frac{\text{one kgf-sec}}{\text{m}^2} = 9.81 \times 100000 \frac{\text{dyne-sec}}{\text{m}^2} = 9.81 \times 100000 \frac{\text{dyne-sec}}{100 \times 100 \times \text{cm}^2}$$

$$= 98.1 \frac{\text{dyne-sec}}{\text{cm}^2} = 98.1 \text{ Poise} \qquad \left\{\because \ \frac{\text{dyne-sec}}{\text{cm}^2} = \text{Poise}\right\}$$

Thus for solving numerical problems, if viscosity is given in poise, it must be divided by 98.1 to get its equivalent numerical value in MKS.

But $\dfrac{\text{one kgf-sec}}{\text{m}^2} = \dfrac{9.81 \text{ N s}}{\text{m}^2} = 98.1 \text{ poise}$

$$\therefore \quad \frac{\text{one N s}}{\text{m}^2} = \frac{98.1}{9.81} \text{ poise} = 10 \text{ poise} \quad \text{or} \quad \text{One Poise} = \frac{1}{10} \frac{\text{N s}}{\text{m}^2}.$$

Alternate Method. One Poise $= \dfrac{\text{dyne} \times \text{s}}{\text{cm}^2} = \left(\dfrac{1 \text{ gm} \times 1 \text{ cm}}{\text{s}^2}\right) \times \dfrac{\text{s}}{\text{cm}^2}$

But dyne $= 1 \text{ gm} \times \dfrac{1 \text{ cm}}{\text{s}^2}$

$$\therefore \quad \text{One Poise} = \frac{1 \text{ gm}}{\text{s cm}} = \frac{\frac{1}{1000} \text{ kg}}{\text{s} \frac{1}{100} \text{ m}}$$

$$= \frac{1}{1000} \times 100 \frac{\text{kg}}{\text{s m}} = \frac{1}{10} \frac{\text{kg}}{\text{s m}} \quad \text{or} \quad 1 \frac{\text{kg}}{\text{s m}} = 10 \text{ poise}.$$

Note. (*i*) In SI units second is represented by 's' and not by 'sec'.

(*ii*) If viscosity is given in poise, it must be divided by 10 to get its equivalent numerical value in SI units.

Sometimes a unit of viscosity as centipoise is used where

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise} \quad \text{or} \quad 1 \text{ cP} = \frac{1}{100} \text{ P} \qquad [\text{cP} = \text{Centipoise, P} = \text{Poise}]$$

The viscosity of water at 20°C is 0.01 poise or 1.0 centipoise.

1.3.2. Kinematic Viscosity. It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the Greek symbol (ν) called '*nu*'. Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \quad \text{...(1.4)}$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \dfrac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\dfrac{\text{Mass}}{\text{Length}}}$$

$$= \frac{\text{Mass} \times \dfrac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\dfrac{\text{Mass}}{\text{Length}}\right)} \quad \left\{ \because \text{Force} = \text{Mass} \times \text{Acc.} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \right\}$$

$$= \frac{(\text{Length})^2}{\text{Time}}.$$

In MKS and SI, the unit of kinematic viscosity is metre^2/sec or m^2/sec while in CGS units it is written as cm^2/s. In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= \text{cm}^2/\text{s} = \left(\frac{1}{100}\right)^2 \text{m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$

Centistoke means $= \frac{1}{100}$ stoke.

1.3.3. Newton's Law of Viscosity. It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity. Mathematically, it is expressed as given by equation (1.2) or as

$$\tau = \mu \frac{du}{dy}.$$

Fluids which obey the above relation are known as **Newtonian** fluids and the fluids which do not obey the above relation are called **Non-newtonian** fluids.

1.3.4. Variation of Viscosity with Temperature. Temperature affects the viscosity. The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with the increase of temperature. This is due to reason that in liquids the cohesive forces predominates the molecular momentum transfer, due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity. But in case of gases the cohesive force are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids gases are :

(*i*) **For liquids,** $\mu = \mu_0 \left(\frac{1}{1 + \alpha t + \beta t^2} \right)$

where μ = Viscosity of liquid at t°C, in poise

μ_0 = Viscosity of liquid at 0°C, in poise

α, β = are constants for the liquid

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = .000221$.

(*ii*) **For a gas,** $\mu = \mu_0 + \alpha t - \beta t^2$

where for air $\mu_0 = .000017$, $\alpha = .000000056$, $\beta = .1189 \times 10^{-9}$.

1.3.5. Types of Fluids. The fluids may be classified into the following five types :

1. Ideal fluid,
2. Real fluid,
3. Newtonian fluid,
4. Non-Newtonian fluid, and
5. Ideal plastic fluid.

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

2. Real Fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids, in actual practice, are real fluids.

3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.

4. Non-Newtonian Fluid. A real fluid, in which the shear stress is not proportional to the rate of shear strain (or velocity gradient), is known as a Non-Newtonian fluid.

5. Ideal Plastic Fluid. A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

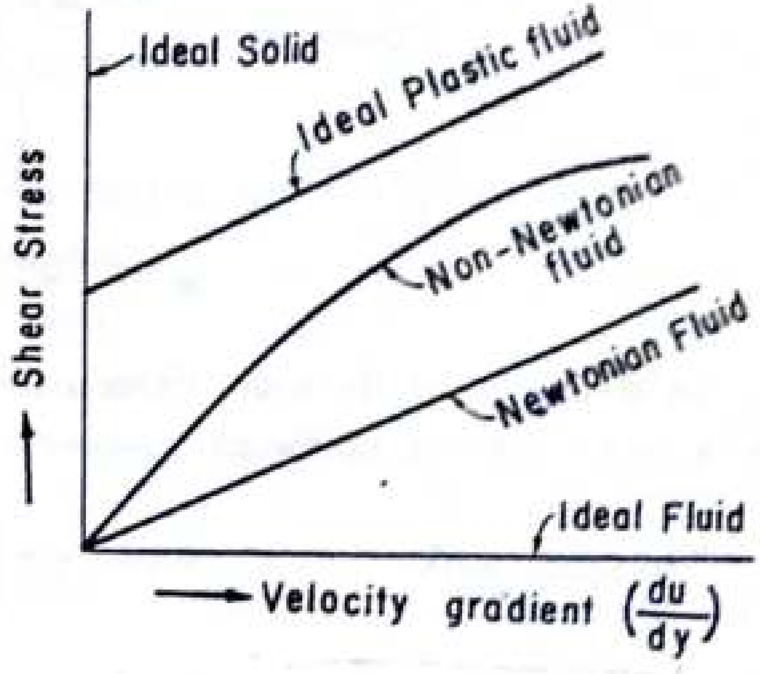


Fig. 1.2. Types of fluids.

1.6. SURFACE TENSION AND CAPILLARITY

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrance under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

The phenomenon of surface tension is explained by Fig. 1.10. Consider three molecules *A, B, C* of a liquid in a mass of liquid. The molecule *A* is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule *A* is zero. But the molecule *B*, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule *B* is acting in the downward direction. The molecule *C*, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid acts as though it is an elastic membrance under tension.

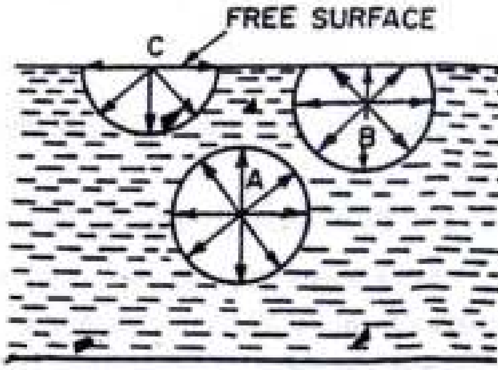


Fig. 1.10. Surface tension.

1.6.1. Surface Tension on Liquid Droplet. Consider a small spherical droplet of a liquid of radius '*r*'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity)

d = Dia. of droplet.

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(*i*) tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 1.11 (*b*) and this is equal to

$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d$$

(*ii*) pressure force on the area $\frac{\pi}{4}d^2$ and $= p \times \frac{\pi}{4}d^2$ as shown in Fig. 1.11 (*c*). These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

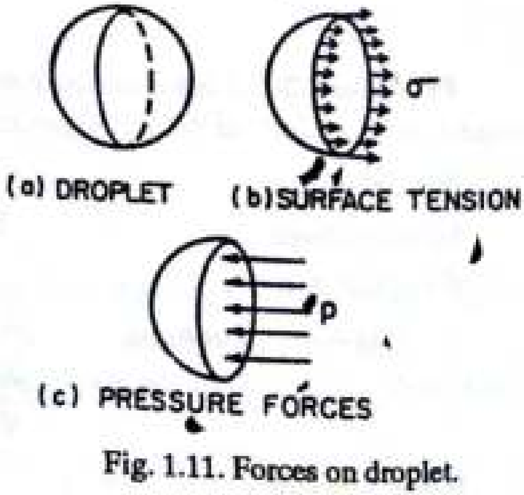


Fig. 1.11. Forces on droplet.

$$p \times \frac{\pi}{4}d^2 = \sigma \times \pi d$$

or

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4}d^2} = \frac{4\sigma}{d} \quad \text{...(1.14)}$$

Equation (1.14) shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

1.6.2. Surface Tension on a Hollow Bubble. A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

$\therefore$ $$p = \frac{2\sigma\pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \quad \text{...(1.15)}$$

1.6.3. Surface Tension on a Liquid Jet. Consider a liquid jet of diameter '*d*' and length '*L*' as shown in Fig. 1.12.

Let p = Pressure intensity inside the liquid jet above the outside pressure

σ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

Force due to pressure $= p \times$ area of semi jet

$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L$.

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$\therefore$ $$p = \frac{\sigma \times 2L}{L \times d} = \frac{2\sigma}{d} \quad \text{...(1.16)}$$

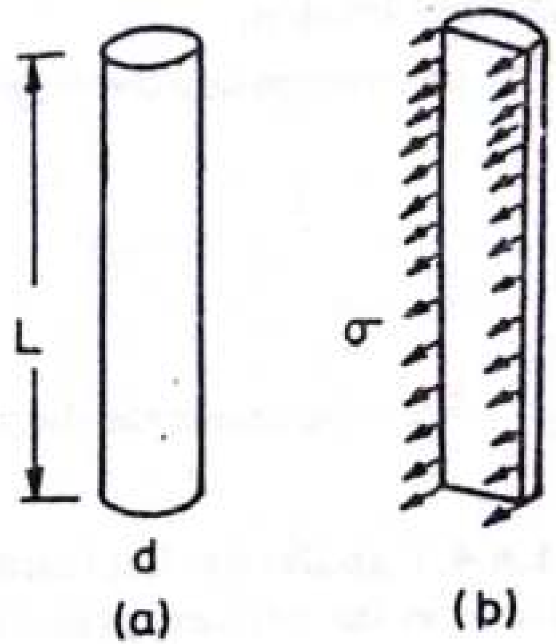


Fig. 1.12. Forces on liquid jet.

1.6.4. Capillarity. Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Expression for Capillary Rise. Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

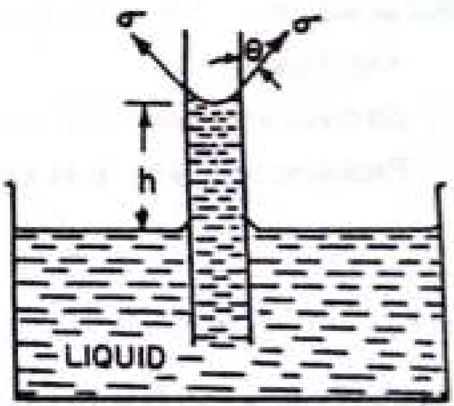


Fig. 1.13. Capillary rise.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube

$$= (\text{Area of tube} \times h) \times \rho \times g$$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad \text{...(1.17)}$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta \quad \text{...(1.18)}$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d} \quad \text{...(1.19)}$$

The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \qquad \text{...(1.20)}$$

Expression for Capillary Fall. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig. 1.14.

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one, is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos \theta$.

Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2$$

$$= \rho g \times h \times \frac{\pi}{4} d^2 \qquad \{\because \; p = \rho g h\}$$

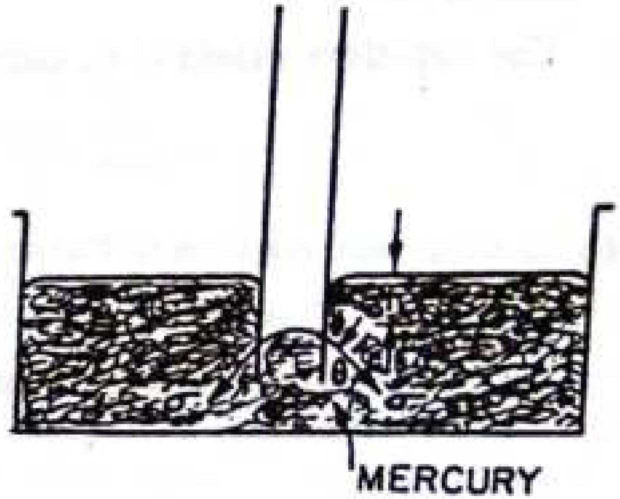


Fig. 1.14

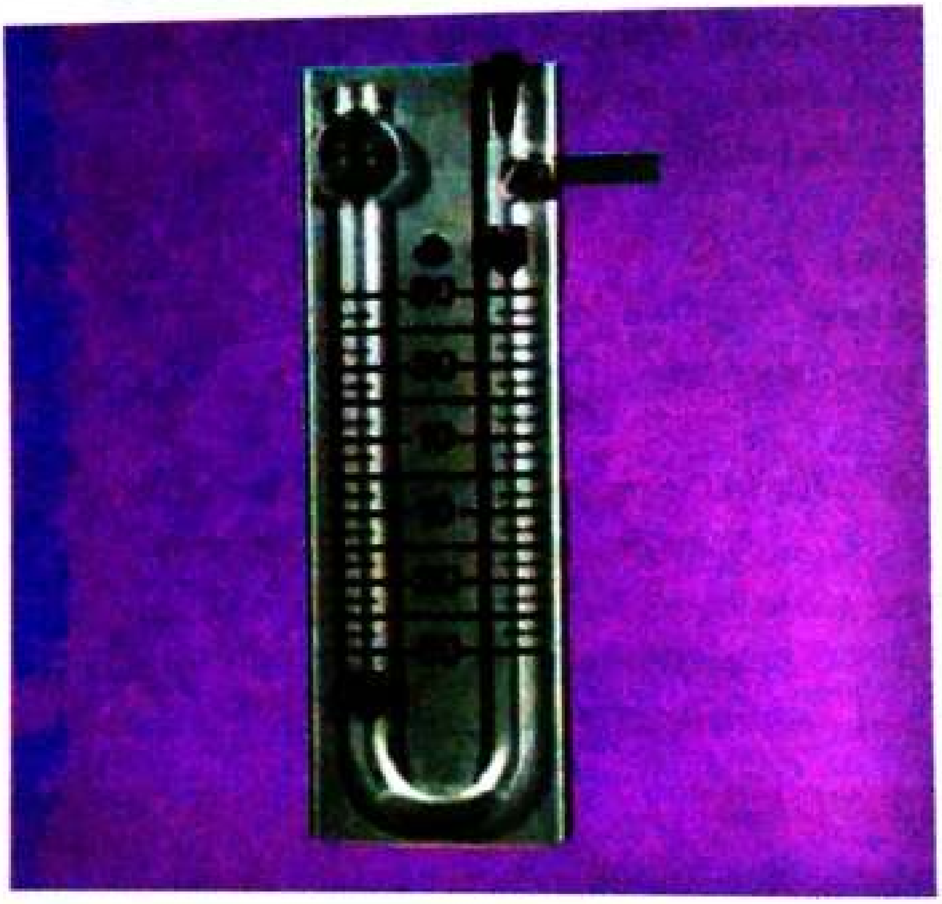
Equating the two, we get

$$\sigma \times \pi d \times \cos \theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \qquad h = \frac{4\sigma \cos \theta}{\rho g d} \qquad \text{...(1.21)}$$

Value of θ for mercury and glass tube is 128°.

Fluid Pressure
its Measureme



2.1. Fluid Pressure

We see that whenever a liquid (such as water, oil etc.) is contained in a vessel, it exerts force at all points on the sides and bottom of the container. This force per unit area is called **pressure**. If P is the force acting on area a, then *Intensity of pressure,

$$p = \frac{P}{a} \quad , \quad P = \frac{F}{A}$$

The direction of this pressure is always at right angles to the surface, with which the fluid at rest, comes in contact.

[***Note:** The intensity of pressure, in brief, is generally termed as *pressure*.]

2.2. Pressure Head

Consider a vessel containing some liquid as shown in Fig. 2.1. We know that, the liquid will exert pressure on all sides as well as bottom of the vessel. Now, let a bottomless cylinder be made to stand in the liquid as shown in the figure.

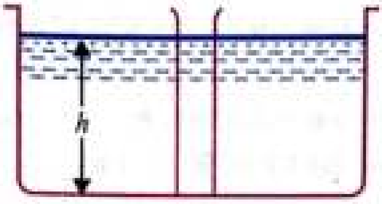


Fig. 2.1. Pressure head.

Let w = Specific weight of the liquid,

h = Height of liquid in the cylinder, and

A = Area of the cylinder base.

A little consideration will show that, there will be some pressure on the cylinder base due to weight of the liquid in it. Therefore, Pressure,

$$p = \frac{\text{Weight of liquid in the cylinder}}{\text{Area of the cylinder base}} = \frac{mg}{A} = \frac{V\rho g}{A}$$

$$= \frac{whA}{A} = wh = h\rho g \qquad = \frac{A \times h \times \rho \times g}{A} = h\rho g$$

This equation shows that the intensity of pressure at any point, in a liquid, is proportional to its depth, from the surface (as w is constant for the given liquid). It is thus obvious that, the pressure can be expressed in either of the following two ways :

$$\boxed{P = h\rho g}$$

1. As a force per unit area *i.e.*, N/m^2, kN/m^2 etc.
2. As a height of the equivalent liquid column.

Note : The pressure is always expressed in pascal (briefly written as Pa) such that 1 Pa = 1 N/m^2, 1 kPa = 1 kN/m^2 and 1 MPa = 1 MN/m^2 = 1 N/mm^2.

Example 2.1. *Find the pressure at a point 4 m below the free surface of water.*

Solution. Given : h = 4 m.

We know that, Pressure at the point,

$$p = wh = 9.81 \times 4 = 39.24 \text{ kN/m}^2 = 39.24 \text{ kPa} \quad \textbf{Ans.}$$

$\times 1000 = 39240 \text{ N/m}^2$

Example 2.2. *A steel plate is immersed in an oil of specific weight 7.5 kN/m³ upto a depth of 2.5 m. What is the intensity of pressure on the plate due to the oil ?*

Solution. Given : $w = 7.5 \text{ kN/m}^3$ and h = 2.5 m.

We know that, Intensity of pressure on the plate,

$$p = wh = 7.5 \times 2.5 = 18.75 \text{ kN/m}^2 = 18.75 \text{ kPa} \quad \textbf{Ans.}$$

Example 2.3. *Calculate the height of a water column equivalent to a pressure of 0.15 MPa.*

Solution. Given : $p = 0.15 \text{ MPa} = 0.15 \times 10^3 \text{ kN/m}^2$

Let h = Height of water column in metres.

We know that, Pressure of water column (p),

$$0.15 \times 10^3 = wh = 9.81 \times h$$

or $$h = (0.15 \times 10^3)/9.81 = 15.3 \text{ m} \quad \textbf{Ans.}$$

$P = h\rho g \Rightarrow 0.15 \times 10^6 = h \times \rho \times g$

$\Rightarrow 0.15 \times 10^6 = h \times 1000 \times 9.81$

$\Rightarrow h = 15.3 \text{ m}$

2.2. PASCAL'S LAW

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element is unity and p_x, p_y and p_z are the pressures or intensity of pressure acting on the face AB, AC and BC respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

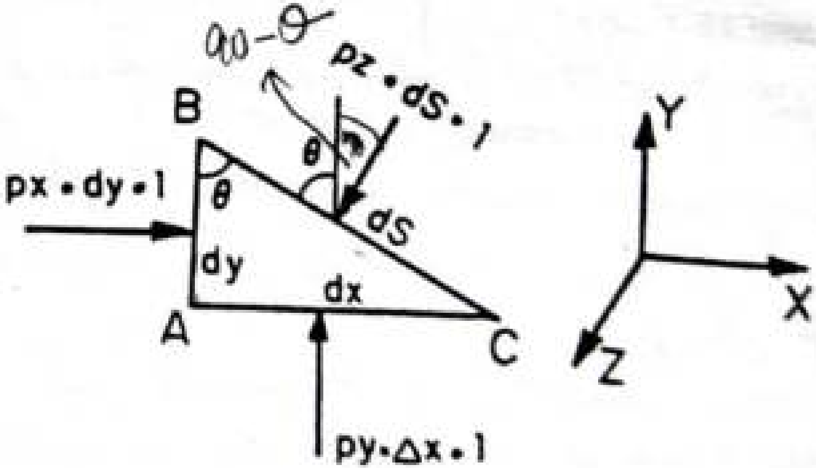


Fig. 2.1. Forces on a fluid element.

1. Pressure forces normal to the surfaces.
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face AB $= p_x \times$ Area of face AB

$= p_x \times dy \times 1$

Similarly force on the face AC $= p_y \times \Delta x \times 1$

Force on the face BC $= p_z \times ds \times 1$

Weight of element $= \dfrac{AB \times AC}{2} \times 1 \times w$, where w = weight density of fluid.

Resolving the forces in x-direction, we have

$$p_x \times dy \times 1 - p_z (ds \times 1) \sin (90° - \theta) = 0$$

or

$$p_x \times dy \times 1 - p_z ds \times 1 \cos \theta = 0.$$

But from Fig. 2.1, $\quad ds \cos \theta = AB = dy$

$$\therefore \quad p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or

$$p_x = p_z. \qquad ...(2.1)$$

Similarly, resolving the forces in y-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos (90° - \theta) - \frac{dx \times dy}{2} \times 1 \times w = 0$$

or

$$p_y \times dx - p_z ds \sin \theta - \frac{dxdy}{2} \times w = 0.$$

But $ds \sin \theta = dx$ and also the element is very small and hence weight is negligible.

$$\therefore \quad p_y dx - p_z \times dx = 0$$

or

$$p_y = p_z \qquad ...(2.2)$$

From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \qquad ...(2.3)$$

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

Problem 2.3. *Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000$ kg/m^3.*

Sol. Given :

Height of liquid column, $Z = 0.3$ m.

The pressure at any point in a liquid is given by equation (2.5) as

$$p = \rho g Z$$

(*a*) For water, $\rho = 1000$ kg/m^3

$\therefore$ $p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943$ N/m^2

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2}. \quad \textbf{Ans.}$$

(*b*) For oil of sp. gr. 0.8, ρ_0 = Sp. gr. × Density of water

$= 0.8 \times \rho = 0.8 \times 1000 = 800$ kg/m^3

$\therefore$ $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}}. \quad \textbf{Ans.}$$

(*c*) For mercury, sp. gr. $= 13.6$

$\therefore$ Density, $\rho_s = 13.6 \times 1000 = 13600$ kg/m^3

Problem 2.4. *The pressure intensity at a point in a fluid is given 3.924 N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.*

Sol. Given :

Pressure intensity, $p = 3.924 \dfrac{\text{N}}{\text{cm}^2} = 3.924 \times 10^4 \dfrac{\text{N}}{\text{m}^2}$.

The corresponding height, Z, of the fluid is given by equation (2.6) as

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$ $Z = \dfrac{p}{\rho \times g} = \dfrac{3.924 \times 10^4}{1000 \times 9.81} = $ **4 m of water. Ans.**

(*b*) For oil, sp. gr. $= 0.9$

$\therefore$ $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$\therefore$ $Z = \dfrac{p}{\rho_0 \times g} = \dfrac{3.924 \times 10^4}{900 \times 9.81} = $ **4.44 m of oil. Ans.**

Problem 2.5. *An oil of sp. gr. 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at the point.*

Sol. Given :

Sp. gr. of oil, $S_0 = 0.9$

Height of oil, $Z_0 = 40 \text{ m}$

Density of oil, $\rho_0 = 1000 \times S_0 = 1000 \times 0.9 = 900 \text{ kg/m}^3$

Intensity of pressure, $p = \rho_0 \times g \times Z_0 = 900 \times 9.81 \times 40 \dfrac{\text{N}}{\text{m}^2}$

$\therefore$ Corresponding height of water $= \dfrac{p}{\text{Density of water} \times g}$

$$= \frac{900 \times 9.81 \times 40}{1000 \times 9.81} = 0.9 \times 40 = \textbf{36 m of water. Ans.}$$

Problem 2.6. *An open tank contains water upto a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1 m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.*

Sol. Given :

Height of water, $Z_1 = 2 \text{ m}$

Height of oil, $Z_2 = 1 \text{ m}$

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000 \text{ kg/m}^3$

Density of oil, $\rho_2 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Pressure intensity at any point is given by

$$p = \rho \times g \times Z.$$

(*i*) At interface, *i.e.*, at *A*

$$p = \rho_2 \times g \times 1.0$$

$$= 900 \times 9.81 \times 1.0$$

$$= 8829 \frac{N}{m^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2.} \quad \textbf{Ans.}$$

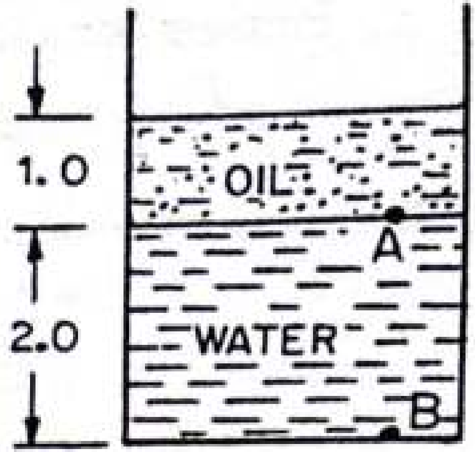


Fig. 2.4

(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1 = 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449\ N/m^2 = \frac{28449}{10^4}\ N/cm^2 = \mathbf{2.8449\ N/cm^2.} \quad \textbf{Ans.}$$

2.4. ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. **Absolute Pressure** is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. **Gauge pressure** is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. **Vacuum Pressure** is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{abs} = p_{atm} + p_{Gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure – Absolute pressure.

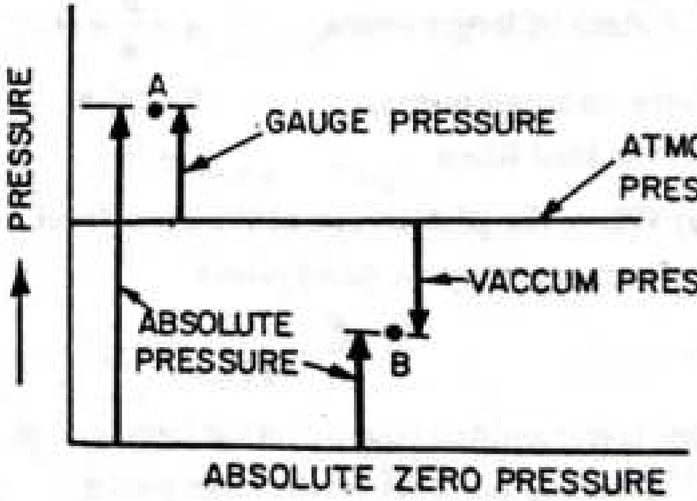


Fig. 2.7. Relationship between pressures.

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case of MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem 2.8. *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of* 1.53×10^3 *kg/m*^{3} *if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m*^{3}.

(A.M.I.E., Summer 1986)

Sol. Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

$\therefore$ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = density of Hg = 13.6×1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

$$\therefore \quad p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \qquad (\because \; Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

$\therefore$ Gauge pressure, p = **45028 N/m^2. Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

2.5. MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers
2. Mechanical Gauges.

2.5.1. Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2. Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,

(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

2.6. SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1. Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \frac{\text{N}}{\text{m}^2}.$$

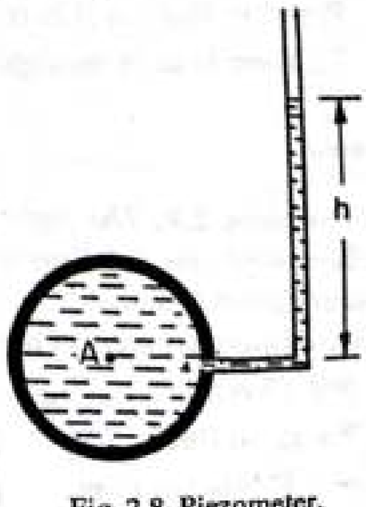


Fig. 2.8. Piezometer.

2.6.2. U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

(*a*) For Gauge pressure. (*b*) For Vacuum pressure.

Fig. 2.9. U-tube Manometer.

(*a*) For Gauge Pressure. Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = Height of light liquid above the datum line

h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = Density of light liquid = $1000 \times S_1$

S_2 = Sp. gr. of heavy liquid

ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1).$$

(*b*) For Vacuum Pressure. For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A = 0

$$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1). \qquad ...(2.8)$$

Problem 2.9. *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Sol. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

Sp. gr. of mercury, $S_2 = 13.6$

∴ Density of mercury, $\rho_2 = 13.6 \times 1000 \text{ kg/m}^3$

Difference of mercury level $h_2 = 20 \text{ cm} = 0.2 \text{ m}$

Height of fluid from *A-A*, $h_1 = 20 - 12 = 8 \text{ cm} = 0.08 \text{ m}$

Let p = Pressure of fluid in pipe

Equating the pressure above *A-A*, we get

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$$= 26683 - 706 = 25977 \text{ N/m}^2 = \mathbf{2.597 \text{ N/cm}^2}. \quad \textbf{Ans.}$$

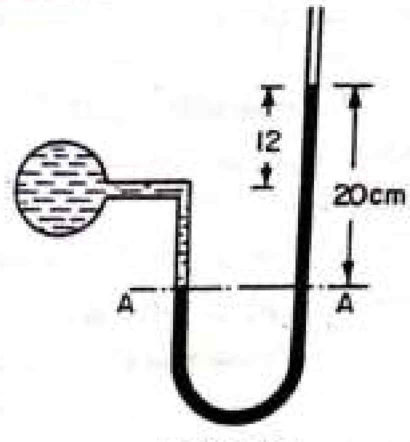


Fig. 2.10

Problem 2.10. *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Sol. Given :

Sp. gr. of fluid, $S_1 = 0.8,$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Difference of mercury level, $h_2 = 40 \text{ cm} = 0.4 \text{ m}$. Height of liquid in left limb, $h_1 = 15 \text{ cm} = 0.15 \text{ m}$. Let the pressure in pipe $= p$. Equating pressure above datum line *A-A*, we get

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

∴ $$p = -[\rho_2 g h_2 + \rho_1 g h_1]$$

$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$

$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2}. \quad \textbf{Ans.}$$

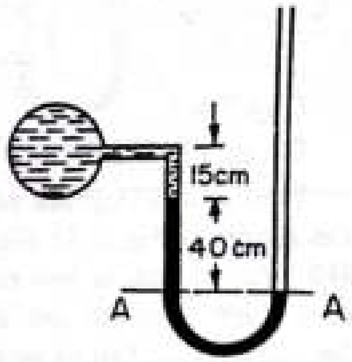


Fig. 2.11

Problem 2.11. *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.* (A.M.I.E., Winter 1989)

Sol. Given :

Difference of mercury $= 10 \text{ cm} = 0.1 \text{ m}$

The arrangement is shown in Fig. 2.11 (*a*)

Let p_A = pressure of water in pipe line (*i.e.*, at point *A*)

The points *B* and *C* lie on the same horizontal line. Hence pressure at *B* should be equal to pressure at *C*. But pressure at *B*.

$$= \text{pressure at } A + \text{pressure due to 10 cm (or 0.1 m) of water}$$

$$= p_A + \rho \times g \times h$$

where $\rho = 1000 \text{ kg/m}^3$ and $h = 0.1 \text{ m}$

$$= p_A + 1000 \times 9.81 \times 0.1$$
$$= p_A + 981 \text{ N/m}^2 \quad \ldots(i)$$

Pressure at C = pressure at D + pressure due to 10 cm of mercury

$$= 0 + \rho_0 \times g \times h_0$$

where ρ_0 for mercury = 13.6×1000 kg/m^3

and h_0 = 10 cm = 0.1 m

$\therefore$ Pressure at C $= 0 + (13.6 \times 1000) \times 9.81 \times 0.1$
$= 13341.6$ N ...(ii)

But pressure at B is equal to pressure at C. Hence equating the equations (i) and (ii), we get

$$p_A + 981 = 13341.6$$

$$\therefore \quad p_A = 13341.6 - 981$$
$$= 12360.6 \frac{\text{N}}{\text{m}^2}. \quad \text{Ans.}$$

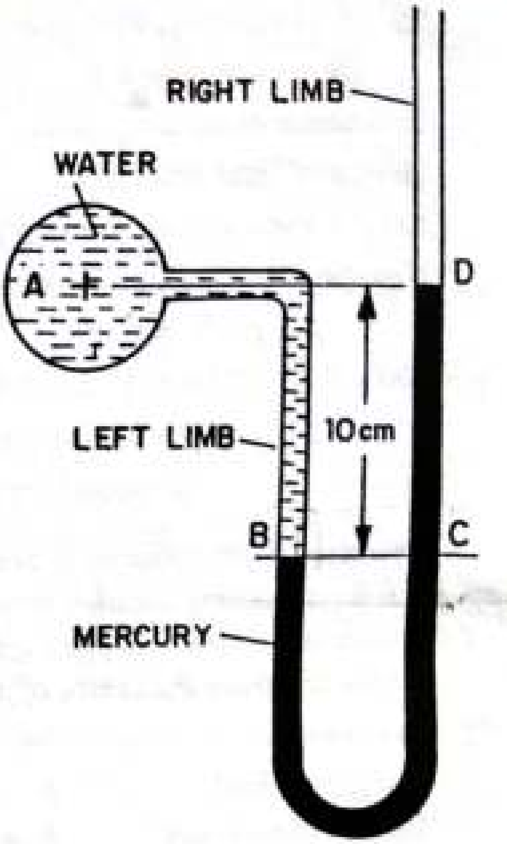


Fig. 2.11 (a)

IInd Part

Given, $p_A = 9810$ N/m^2

Find new difference of mercury level. The arrangement is shown in Fig. 2.11 (b). In this case the pressure at A is 9810 N/m^2 which is less than the 12360.6 N/m^2. Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x = Rise of mercury in left limb in cm

Then fall of mercury in right limb = x cm

The points B, C and D show the initial conditions whereas points B^*, C^* and D^* show the final conditions.

The pressure at B^* = pressure at C^*

or Pressure at A + pressure due to $(10 - x)$ cm of water = pressure at D^* + pressure due to $(10 - 2x)$ cm of mercury

or $p_A + \rho_1 \times g \times h_1 = p_D{}^* + \rho_2 \times g \times h_2$

or $9810 + 1000 \times 9.81 \times \left(\frac{10 - x}{100}\right)$

$$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10 - 2x}{100}\right)$$

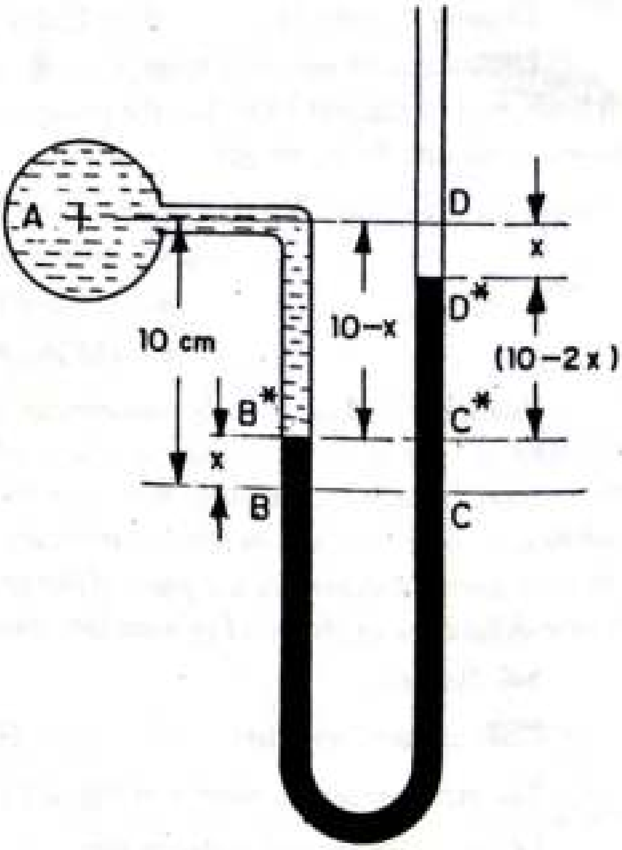


Fig. 2.11 (b)

Dividing by 9.81, we get

$$1000 + 100 - 10x = 1360 - 272x$$

or
$$272x - 10x = 1360 - 1100$$

or
$$262x = 260$$

or

$$\therefore \qquad x = \frac{260}{262} = 0.992 \text{ cm}$$

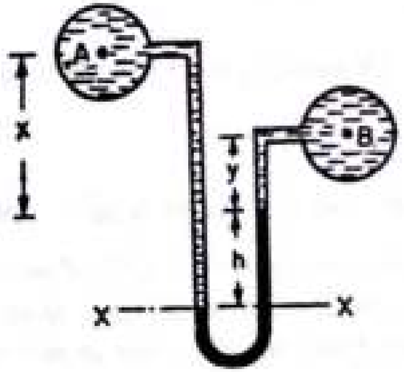
$\therefore$ New difference of mercury $= 10 - 2x$ cm $= 10 - 2 \times 0.992 =$ **8.016 cm. Ans.**

2.7. DIFFERENTIAL MANOMETERS

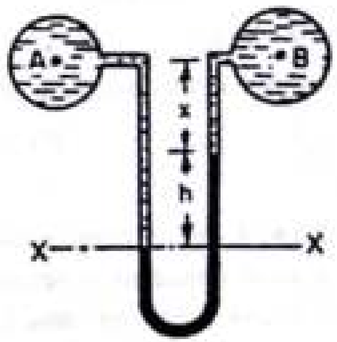
Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

2.7.1. U-tube Differential Manometer. Fig. 2.18 shows the differential manometers of U-tube type.



(*a*) Two pipes at different levels.



(*b*) *A* and *B* are at the same level.

Fig. 2.18. U-tube differential manometers.

Fig. 2.18. (*a*). Let the two points *A* and *B* are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at *A* and *B* are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of *B*, from the mercury level in the right limb.

x = Distance of the centre of *A*, from the mercury level in the right limb.

ρ_1 = Density of liquid at *A*.

ρ_2 = Density of liquid at *B*.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at *X-X*.

Pressure above *X-X* in the left limb = $\rho_1 g(h + x) + p_A$

where p_A = pressure at *A*.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx \qquad ...(2.12)$$

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

[Fig. 2.18. (*b*). A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1)]. \qquad ...(2.13)$$

Problem 2.15. *A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.*

Sol. Given :

Sp. gr. of oil, $S_1 = 0.9$ $\therefore$ Density, $\rho_1 = 0.9 \times 1000 = 900$ kg/m^3

Difference in mercury level, $h = 15$ cm $= 0.15$ m

Sp. gr. of mercury, $S_g = 13.6$ $\therefore$ Density, $\rho_g = 13.6 \times 1000$ kg/m^3

The difference of pressure is given by equation (2.13)

or $$p_A - p_B = g \times h(\rho_g - \rho_1)$$

$= 9.81 \times 0.15\ (13600 - 900) =$ **18688 N/m^2. Ans.**

Problem 2.16. *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. Find the difference in mercury level in the differential manometer.*

Sol. Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ $\therefore$ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ $\therefore$ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm^2 $= 1 \times 10^4$ kgf/m^2

$= 10^4 \times 9.81$ N/m^2 ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm^2

$= 1.8 \times 10^4$ kgf/m^2

$= 1.8 \times 10^4 \times 9.81$ N/m^2 ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m^3

Taking X-X as datum line.

Pressure above X-X in the left limb

$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$

$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$

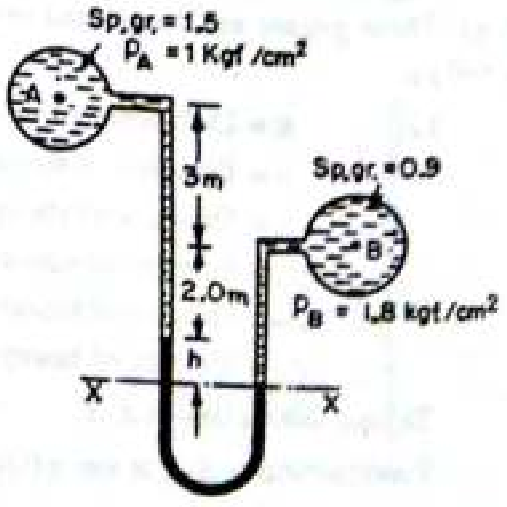


Fig. 2.19

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4 = 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

or $$(13.6 - 0.9)h = 19.8 - 17.5 \quad \text{or} \quad 12.7h = 2.3$$

$\therefore$ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1 \text{ cm.}} \quad \textbf{Ans.}$$

Problem 2.17. *A differential manometer is connected at the two points A and B as shown in Fig. 2.20. At B air pressure is 9.81 N/cm² (abs), find the absolute pressure at A.*

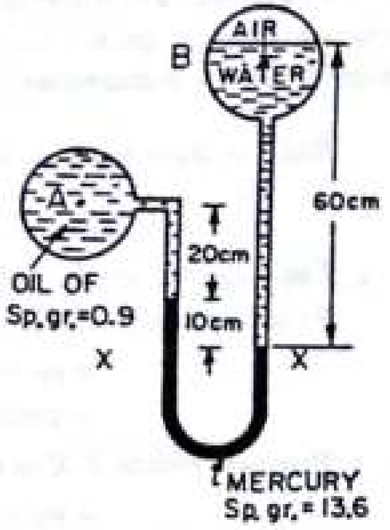


Fig. 2.20

Sol. Air pressure at B $= 9.81$ N/cm²

or $p_B = 9.81 \times 10^4$ N/m²

Density of oil $= 0.9 \times 1000 = 900$ kg/m³

Density of mercury $= 13.6 \times 1000$ kg/m³

Let the pressure at A is p_A

Taking datum line at X-X

Pressure above X-X in the right limb

$= 1000 \times 9.81 \times 0.6 + p_B$

$= 5886 + 98100 = 103986$

Pressure above X-X in the left limb

$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$

$= 13341.6 + 1765.8 + p_A$

Equating the two pressure head

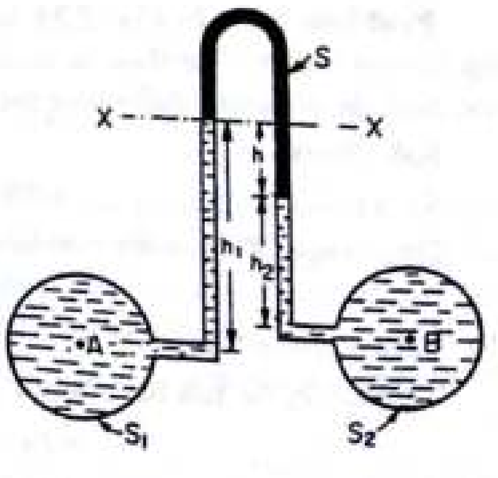
$$103986 = 13341.6 + 1765.8 + p_A$$

$\therefore$ $$p_A = 103986 - 15107.4 = 88876.8$$

$\therefore$ $$p_A = 88876.8 \text{ N/m}^2 = \frac{88876.8 \text{ N}}{10000 \text{ cm}^2} = 8.887 \frac{\text{N}}{\text{cm}^2}.$$

$\therefore$ Absolute pressure at A = **8.887 N/cm².** **Ans.**

2.7.2. Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.



Let h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X.

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or $$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

Problem 2.18. *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

Sol. Pressure head at $A = \frac{p_A}{\rho g} = 2$ m of water

$\therefore$ $$p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$$

Fig. 2.22 shows the arrangement. Taking X-X as datum line.

Pressure below X-X in the left limb

$$= p_A - \rho_1 \times g \times h_1$$
$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$$

Pressure below X-X in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$
$$= p_B - 981 - 941.76$$
$$= p_B - 1922.76$$

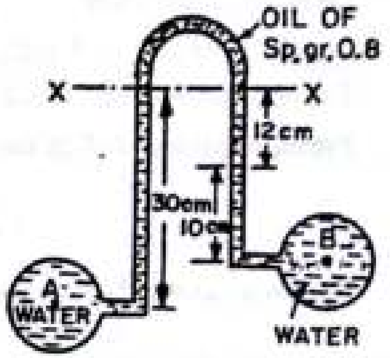


Fig. 2.22

Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

or $$p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$$

or $$p_B = \mathbf{1.8599 \text{ N/cm}^2}. \quad \textbf{Ans.}$$

Problem 2.19. *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

Sol. Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800 \text{ kg/m}^3$

Difference of oil in the two limbs

$$= (30 + 20) - 30 = 20 \text{ cm}$$

Taking datum line at X-X

Pressure in the left limb below X-X

$$= p_A - 1000 \times 9.81 \times 0.$$
$$= p_A - 2943$$

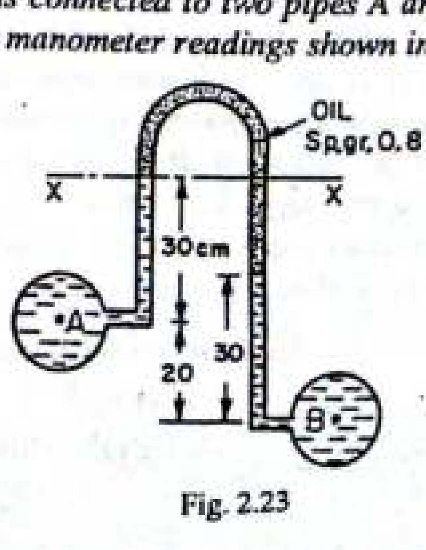


Fig. 2.23

Pressure in the right limb below X-X $= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= p_B - 2943 - 1569.6 = p_B - 4512.6$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore$ $p_B - p_A = 4512.6 - 2943 =$ **1569.6 N/m²**. **Ans.**

Problem 2.20. *Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in Fig. 2.24 below, conveying liquids of specific gravities 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal.* (A.M.I.E., Winter 1985)

Sol. Given :

Fig. 2.24 shows the arrangement. Taking X-X as datum line.

Let p_A = Pressure at A

p_B = Pressure at B

Density of liquid in pipe A

= Sp. gr. × 1000

= 1.2 × 1000

= 1200 kg/m²

Density of liquid in pipe B

= 1 × 1000 = 1000 kg/m³

Density of oil = 0.7 × 1000 = 700 kg/m³

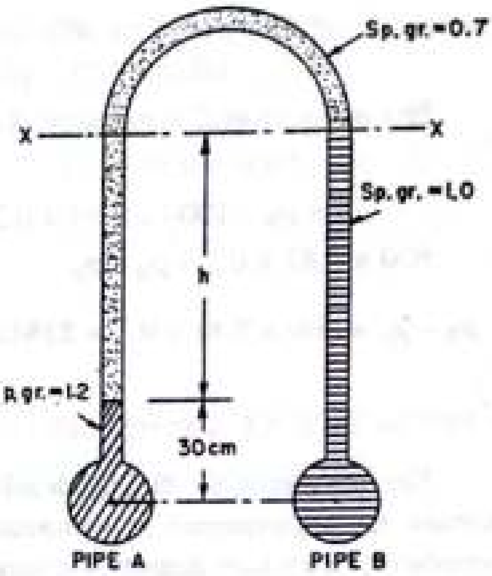


Fig. 2.24

Now pressure below X-X in the left limb.

$= p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h$

Pressure below X-X in the right limb

$= p_B - 1000 \times 9.81 \times (h + 0.3)$

Equating the two pressure, we get

$$p_A - 1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = p_B - 1000 \times 9.81\ (h + 0.3)$$

But $p_A = p_B$ (given)

$\therefore$ $-1200 \times 9.81 \times 0.3 - 700 \times 9.81 \times h = -1000 \times 9.81(h + 0.3)$

Dividing by 1000 × 9.81,

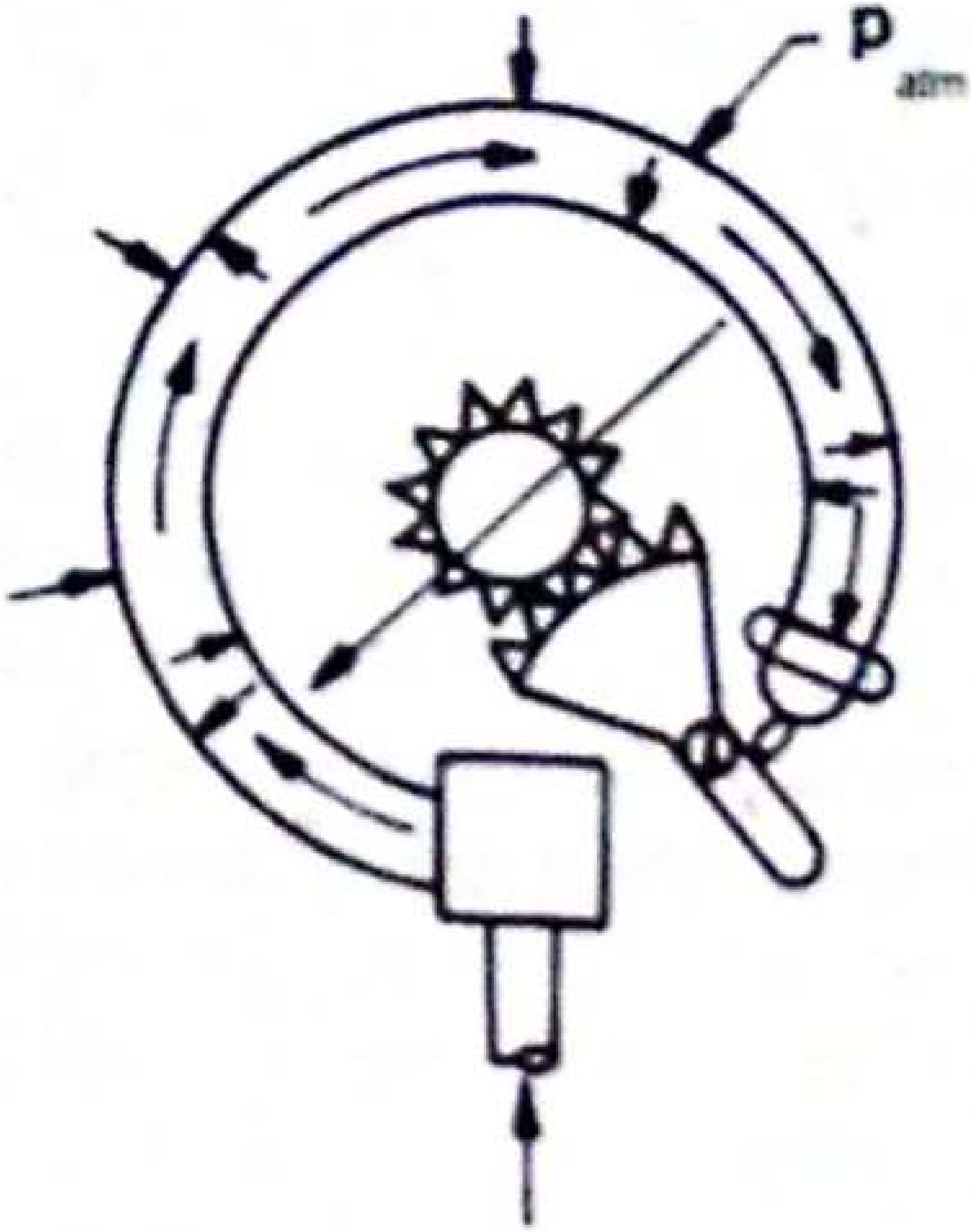
$$-1.2 \times 0.3 - 0.7h = -(h + 0.3)$$

or $0.3 \times 1.2 + 0.7h = h + 0.3$ or $0.36 - 0.3 = h - 0.7h = 0.3h$

$\therefore$
$$h = \frac{0.36 - 0.30}{0.30} = \frac{0.06}{0.30}\ \text{m}$$

$$= \frac{1}{5}\ \text{m} = \frac{1}{5} \times 100 = \textbf{20 cm.}$$ **Ans.**

the Bourdon gauge which measures the difference between the system pressure inside the tube and atmospheric pressure. It relies on the deformation of a bent hollow tube of suitable material which, when subjected to the pressure to be measured on the inside (and atmospheric pressure on the outside), tends to unbend. This moves a pointer through a suitable gear-and-lever mechanism against a calibrated scale. Figure (b) shows an open U-tube indicating gauge pressure, and Fig. (c) shows an open U-tube indicating vacuum. Figure (d) shows a closed U-tube indicating absolute pressure. If p is atmospheric pressure, this is a *barometer*. These are called U-tube manometers.



p
atm
p

3.1. INTRODUCTION

This chapter deals with the fluids (*i.e.*, liquids and gases) at rest. This means that there will be no relative motion between adjacent or neighbouring fluid layers. The velocity gradient, which is equal to the change of velocity between two adjacent fluid layers divided by the distance between the layers, will be zero or $\frac{du}{dy} = 0$. The shear stress which is equal to $\mu \frac{\partial u}{\partial y}$ will also be zero. Then the forces acting on the fluid particles will be :

1. due to pressure of fluid normal to the surface,
2. due to gravity (or self-weight of fluid particles).

3.2. TOTAL PRESSURE AND CENTRE OF PRESSURE

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

3.3. VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F).** The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1.

Pressure intensity on the strip, $p = \rho gh$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho gh \times b \times dh$

∴ Total pressure force on the whole surface,

$$F = \int dF = \int \rho gh \times b \times dh = \rho g \int b \times h \times dh$$

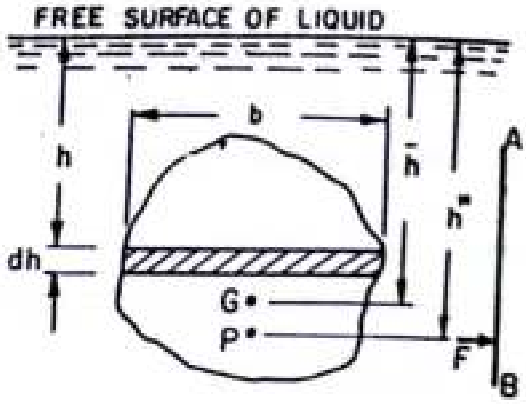


Fig. 3.1

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

∴ $$F = \rho g A \bar{h} \quad \text{...(3.1)}$$

For water the value of $\rho = 1000 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$. The force will be in Newton.

(*b*) **Centre of Pressure (h*).** Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$. ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$= dF \times h$ $\{\because \ dF = \rho gh \times b \times dh\}$

$= \rho gh \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho gh \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int bh^2 \, dh = \rho g \int h^2 dA \quad (\because \ bdh = dA)$$

But $\int h^2 dA = \int bh^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

∴ Sum of moments about free surface

$= \rho g I_0$ ...(3.3)

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But $F = \rho g A \bar{h}$

∴ $$\rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \quad \text{...(3.4)}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_G in equation (3.4), we get

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \quad ...(3.5)$$

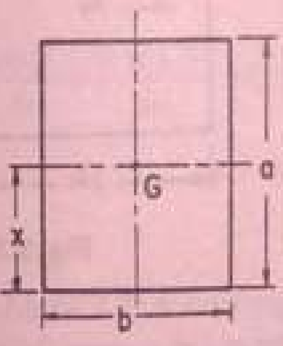
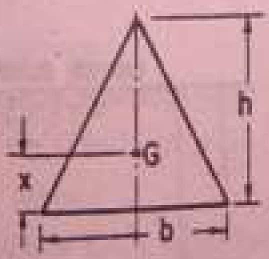
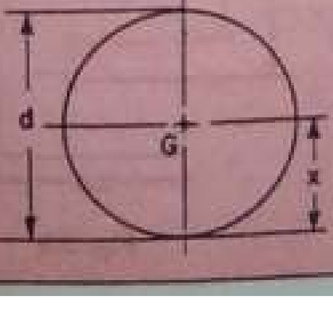
In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

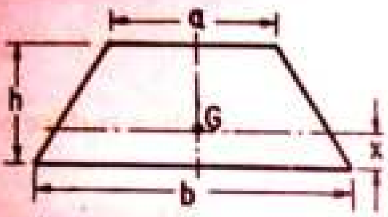
(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

TABLE 3.1

The moments of inertia and other geometric properties of some important plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*	*Moment of inertia about base (I_0)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
4. Trapezium 	$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	—

Problem 3.1. *A rectangular plane surface is 2 m wide and 3 m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincides with water surface, (b) 2.5 m below the free water surface.*

Sol. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

(*a*) **Upper edge coincides with water surface (Fig. 3.2).** Total pressure is given by equation (3.1) as

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3, $g = 9.81$ m/s^2

$A = 3 \times 2 = 6$ m^2, $\bar{h} = \frac{1}{2}(3) = 1.5$ m.

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5$$

$$= \mathbf{88290\ N.} \quad \textbf{Ans.}$$

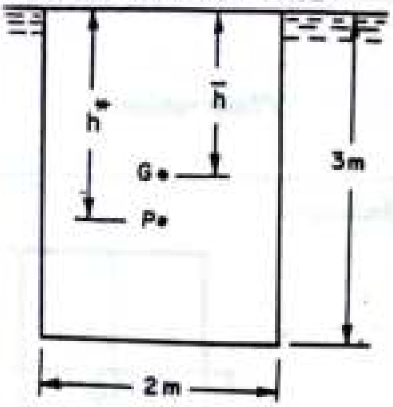


Fig. 3.2

Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0\ m.} \quad \textbf{Ans.}$$

(*b*) **Upper edge is 2.5 m below water surface (Fig. 3.3).** Total pressure (F) is given by (3.1)

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0$$

$$= \mathbf{235440\ N.} \quad \textbf{Ans.}$$

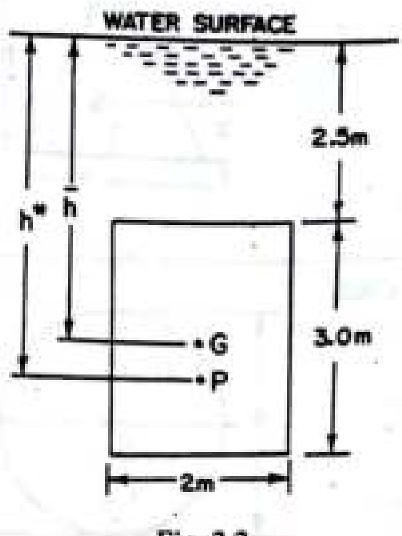


Fig. 3.3

Centre of pressure is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5, A = 6.0, \bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875\ m.} \quad \textbf{Ans.}$$

Problem 3.2. *Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of the plate is 3 m below the free surface of water. Find the position of centre of pressure also.*

Sol. Given : Dia of plate, $d = 1.5$ m

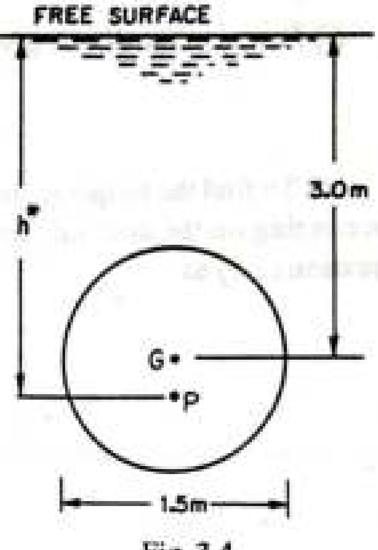


Fig. 3.4

$\therefore$ Area, $A = \frac{\pi}{4}(1.5)^2 = 1.767\ m^2$

$\bar{h} = 3.0$ m

Total pressure is given by equation (3.1),

$$F = \rho g A\bar{h}$$
$$= 1000 \times 9.81 \times 1.767 \times 3.0 \text{ N}$$
$$= \mathbf{52002.81\ N.} \quad \textbf{Ans.}$$

Position of centre of pressure (h^*) is given by equation (3.5)

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi d^4}{64} = \frac{\pi \times 1.5^4}{64} = 0.2485\ m^4$

$$\therefore \quad h^* = \frac{0.2485}{1.767 \times 3.0} + 3.0 = 0.0468 + 3.0$$

$$= \mathbf{3.0468\ m.} \quad \textbf{Ans.}$$

Problem 3.3. *A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to* $\left(p + \frac{d^2}{12p}\right)$.

Sol. Given :

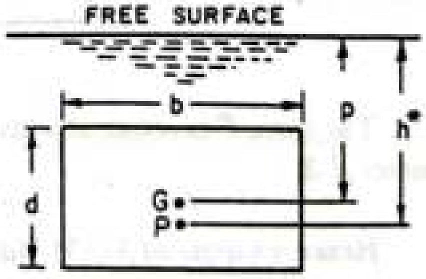


Fig. 3.5

Depth of vertical gate $= d$ m

Let the width of gate $= b$ m

$\therefore$ Area, $A = b \times d\ m^2$

Depth of C.G. from free surface

$\bar{h} = p$ m.

Let h^* is the depth of centre of pressure from free surface, which is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + h, \quad \text{where } I_G = \frac{bd^3}{12}$$

$$\therefore \quad h^* = \left(\frac{bd^3}{12} \Big/ b \times d \times p\right) + p = \frac{d^2}{12p} + p \quad \text{or} \quad \mathbf{p + \frac{d^2}{12}}. \quad \textbf{Ans.}$$

Problem 3.4. *A circular opening, 3 m diameter, in a vertical side of a tank is closed by a disc of 3 m diameter which can rotate about a horizontal diameter. Calculate :*

(i) the force on the disc, and

(ii) the torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4 m. (A.M.I.E., Winter, 1977)

Sol. Given :

Dia of opening, $d = 3$ m

$\therefore$ Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 \text{ m}^2$.

Depth of C.G., $\bar{h} = 4$ m

(*i*) Force on the disc is given by equation (3.1) as

$$F = \rho g A\bar{h}$$

$$= 1000 \times 9.81 \times 7.0685 \times 4.0 = \mathbf{277368\ N = 277.368\ kN.} \quad \textbf{Ans.}$$

(*ii*) To find the torque required to maintain the disc in equilibrium, first calculate the point of application of force acting on the disc, *i.e.*, centre of pressure of the force F. The depth of centre of pressure (h^*) is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0 \qquad \left\{\because\ I_G = \frac{\pi}{64} d^4\right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14 \text{ m}$$

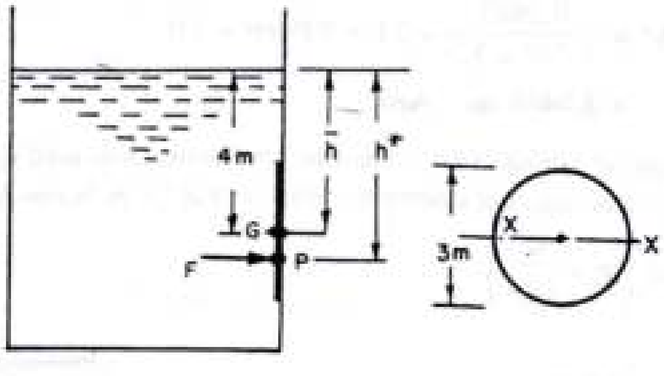


Fig. 3.6

The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

$$= F \times (h^* - \bar{h}) = 277368\ (4.14 - 4.0) = \mathbf{38831\ Nm.} \quad \textbf{Ans.}$$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Problem 3.5. *A pipe line which is 4 m in diameter contains a gate valve. The pressure at the centre of the pipe is 19.6 N/cm². If the pipe is filled with oil of sp. gr. 0.87, find the force exerted by the oil upon the gate and position of centre of pressure.* (Converted to SI Units, A.M.I.E., Winter, 1975)

Sol. Given :

Dia. of pipe, $d = 4$ m

$\therefore$ Area, $A = \frac{\pi}{4} \times 4^2 = 4\pi \text{ m}^2$

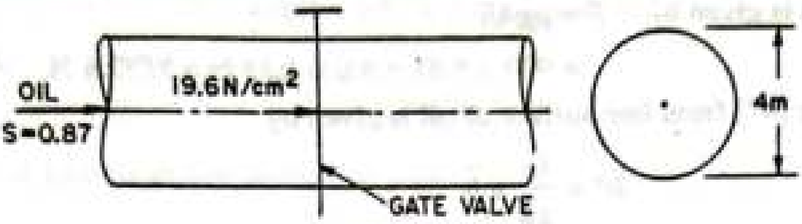


Fig. 3.7

Sp. gr. of oil, $S = 0.87$

∴ Density of oil, $\rho_0 = 0.87 \times 1000 = 870 \text{ kg/m}^3$

∴ Weight density of oil, $w_0 = \rho_0 \times g = 870 \times 9.81 \text{ N/m}^3$

Pressure at the centre of pipe, $p = 19.6 \text{ N/cm}^2 = 19.6 \times 10^4 \text{ N/m}^2$

∴ Pressure head at the centre $= \dfrac{p}{w_0} = \dfrac{19.6 \times 10^4}{870 \times 9.81} = 22.988 \text{ m}$

∴ The height of equivalent free oil surface from the centre of pipe = 22.988 m.

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988$ m

(*i*) Now the force exerted by the oil on the gate is given by

$$F = \rho g A \bar{h}$$

where ρ = density of oil = 870 kg/m³

$$F = 870 \times 9.81 \times 4\pi \times 22.988 = \mathbf{2465500 \text{ N} = 2.465 \text{ MN}}. \quad \textbf{Ans.}$$

(*ii*) Position of centre of pressure (h^*) is given by (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64}d^4}{\frac{\pi}{4}d^2 \times \bar{h}} + \bar{h}$$

$$= \frac{d^2}{16\bar{h}} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988 = 0.043 + 22.988 = \mathbf{23.031 \text{ m}}. \quad \textbf{Ans.}$$

Or centre of pressure is below the centre of the pipe by a distance of 0.043 m. **Ans.**

Problem 3.6. *Determine the total pressure and centre of pressure on an isosceles triangular plate of base 4 m and altitude 4 m when it is immersed vertically in an oil of sp. gr. 0.9. The base of the plate coincides with the free surface of oil.*

Sol. Given :

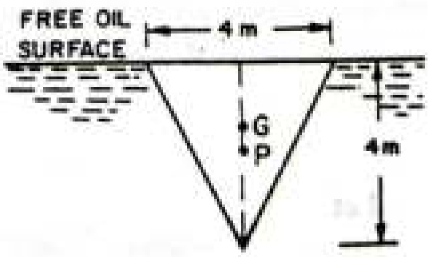


Fig. 3.8

Base of plate, $b = 4$ m

Height of plate, $h = 4$ m

∴ Area, $A = \dfrac{b \times h}{2} = \dfrac{4 \times 4}{2} = 8.0 \text{ m}^2$

Sp. gr. of oil, $S = 0.9$

∴ Density of oil, $\rho = 900 \text{ kg/m}^3$.

The distance of C.G. from free surface of oil,

$$\bar{h} = \frac{1}{3} \times h = \frac{1}{3} \times 4 = 1.33 \text{ m}.$$

Total pressure (F) is given by $F = \rho g A\bar{h}$

$$= 900 \times 9.81 \times 8.0 \times 1.33 \text{ N} = \mathbf{9597.6 \text{ N}}. \quad \textbf{Ans.}$$

Centre of pressure (h^*) from free surface of oil is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. of triangular section about its C.G.

$$= \frac{bh^3}{36} = \frac{4 \times 4^3}{36} = 7.11 \text{ m}^4$$

$$\therefore \quad h^* = \frac{7.11}{8.0 \times 1.33} + 1.33 = 0.6667 + 1.33 = \mathbf{1.99 \text{ m}}. \quad \textbf{Ans.}$$

3.4. HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to, $p = \rho g h$, where h is depth of surface.

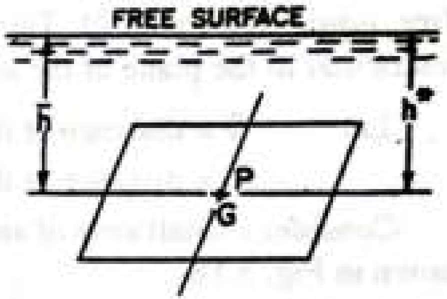


Fig. 3.16

Let A = Total area of surface

Then total force, F, on the surface

$$= p \times \text{Area} = \rho g \times h \times A$$
$$= \rho g A \bar{h}$$

where $\bar{h}$ = Depth of C.G. from free surface of liquid = h

also h^* = Depth of centre of pressure from free surface = h.

Problem 3.13. *Fig. 3.17 shows a tank full of water. Find :*

(i) Total pressure on the bottom of tank.

(ii) Weight of water in the tank.

(iii) Hydrostatic paradox between the results of (i) and (ii) Width of tank is 2 m.

Sol. Given :

Depth of water on bottom of tank

$$h_1 = 3 + 0.6 = 3.6 \text{ m}$$

Width of tank = 2 m

Length of tank at bottom = 4 m

$\therefore$ Area at the bottom, $A = 4 \times 2 = 8 \text{ m}^2$

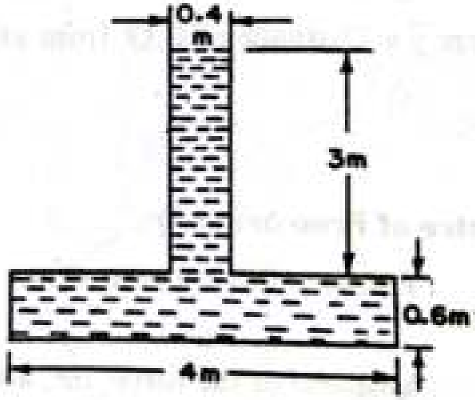


Fig. 3.17

(*i*) Total pressure F, on the bottom is

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 8 \times 3.6$$
$$= \mathbf{282528 \text{ N.}} \quad \textbf{Ans.}$$

(*ii*) Weight of water in tank = ρg × Volume of tank

$$= 1000 \times 9.81 \times [3 \times 0.4 \times 2 + 4 \times .6 \times 2]$$
$$= 1000 \times 9.81 [2.4 + 4.8] = \mathbf{70632 \text{ N.}} \quad \textbf{Ans.}$$

(*iii*) From the results of (*i*) and (*ii*), it is observed that the total weight of water in the tank is much less than the total pressure at the bottom of the tank. This is known as Hydrostatic paradox.

4.1. INTRODUCTION

In this chapter, the equilibrium of the floating and sub-merged bodies will be considered. Thus the chapter will include : 1. Buoyancy, 2. Centre of buoyancy, 3. Metacentre, 4. Metacentric height, 5. Analytical method for determining metacentric height, 6. Conditions of equilibrium of a floating and sub-merged body, and 7. Experimental method for metacentric height.

4.2. BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

4.3. CENTRE OF BUOYANCY

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

Problem 4.1. *Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5 m, when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length 6.0 m.*

Sol. Given :

Width $= 2.5$ m

Depth $= 1.5$ m

Length $= 6.0$ m

Volume of the Block $= 2.5 \times 1.5 \times 6.0 = 22.50 \text{ m}^3$

Density of wood, $\rho = 650 \text{ kg/m}^3$

$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 \text{ N} = 143471 \text{ N}$

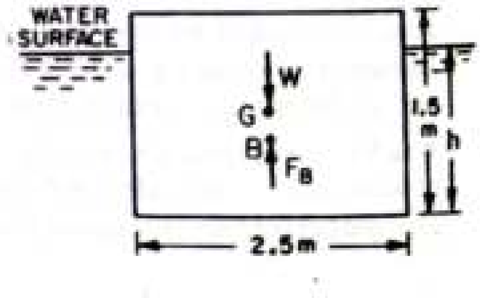


Fig. 4.1

For equilibrium the weight of water displaced = Weight of wooden block

$= 143471$ N

$$\therefore \text{ Volume of water displaced } = \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625 \text{ m}^3}. \text{ Ans.}$$

$(\because$ Weight density of water $= 1000 \times 9.81 \text{ N/m}^3)$

Position of Centre of Buoyancy. Volume of wooden block in water

$= $ Volume of water displaced

$2.5 \times h \times 6.0 = 14.625 \text{ m}^3$ where h is depth of wooden block in water

$$\therefore \qquad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$\therefore$ **Centre of Buoyancy** $= \dfrac{0.975}{2} =$ **0.4875 m from base. Ans.**

Problem 4.3. *A stone weighs 392.4 N in air and 196.2 N in water. Compute the volume of stone and its specific gravity.*

Sol. Given :

Weight of stone in air = 392.4 N

Weight of stone in water = 196.2 N

For equilibrium,

Weight in air – Weight of stone in water = Weight of water displaced

or $392.4 - 196.2 = 196.2 = 1000 \times 9.81 \times$ Volume of water displaced

$\therefore$ Volume of water displaced $= \dfrac{196.2}{1000 \times 9.81} = \dfrac{1}{50}\ \text{m}^3 = \dfrac{1}{50} \times 10^6\ \text{cm}^3 = \mathbf{2 \times 10^4\ cm^3}$. **Ans.**

= Volume of stone

$\therefore$ Volume of stone $= \mathbf{2 \times 10^4\ cm^3}$. **Ans.**

Specific Gravity of Stone

Mass of stone $= \dfrac{\text{Weight in air}}{g} = \dfrac{392.4}{9.81} = 40$ kg

Density of stone $= \dfrac{\text{Mass in air}}{\text{Volume}} = \dfrac{40.0\ \text{kg}}{\dfrac{1}{50}\ \text{m}^3} = 40 \times 50 = 2000\ \dfrac{\text{kg}}{\text{m}^3}$

$\therefore$ Sp. gr. of stone $= \dfrac{\text{Density of stone}}{\text{Density of water}} = \dfrac{2000}{1000} = \mathbf{2.0}$. **Ans.**

Problem 4.4. *A body of dimensions 1.5 m × 1.0 m × 2 m, weighs 1962 N in water. Find its weight in air. What will be its specific gravity ?*

Sol. Volume of body $= 1.50 \times 1.0 \times 2.0 = 3.0\ \text{m}^3$

Weight of body in water = 1962 N

Volume of the water displaced = Volume of the body = $3.0\ \text{m}^3$

$\therefore$ Weight of water displaced $= 1000 \times 9.81 \times 3.0 = 29430$ N

For the equilibrium of the body

Weight of body in air – Weight of water displaced = Weight in water

$\therefore$ $W_{air} - 29430 = 1962$

$W_{air} = 29430 + 1962 = 31392$ N

Mass of body $= \frac{\text{Weight in air}}{g} = \frac{31392}{9.81} = 3200$ kg

Density of the body $= \frac{\text{Mass}}{\text{Volume}} = \frac{3200}{3.0} = 1066.67$

$\therefore$ Sp. gravity of the body $= \frac{1066.67}{1000} = $ **1.067. Ans.**

Problem 4.5. *Find the density of a metallic body which floats at the interface of mercury of sp. gr. 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.*

Sol. Let the volume of the body $= V\ m^3$

Then volume of body sub-merged in mercury

$$= \frac{40}{100} V = 0.4\, V\ m^3$$

Volume of body sub-merged in water

$$= \frac{60}{100} \times V = 0.6\, V\ m^3$$

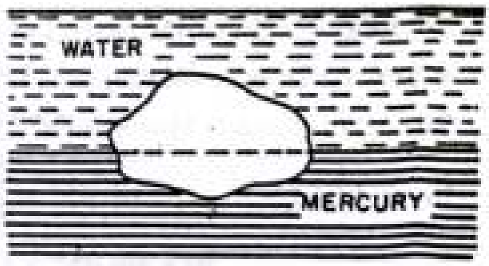


Fig. 4.3

For the equilibrium of the body

Total buoyant force (upward force) = Weight of the body

But total buoyant force = Force of buoyancy due to water + Force of buoyancy due to mercury

Force of buoyancy due to water = Weight of water displaced by body

= Density of water × g × Volume of water displaced

= $1000 \times g \times$ Volume of body in water

= $1000 \times g \times 0.6 \times V$ N

and Force of buoyancy due to mercury = Weight of mercury displaced by body

= g × Density of mercury × Volume of mercury displaced

= $g \times 13.6 \times 1000 \times$ Volume of body in mercury

= $g \times 13.6 \times 1000 \times 0.4\, V$ N

Weight of the body = Density × g × Volume of body $= \rho \times g \times V$

where ρ is the density of the body

$\therefore$ For equilibrium, we have

Total buoyant Force = Weight of the body

$$1000 \times g \times 0.6 \times V + 13.6 \times 1000 \times g \times .4\, V = \rho \times g \times V$$

or $$\rho = 600 + 13600 \times .4 = 600 + 54400 = 6040.00 \text{ kg/m}^3$$

$\therefore$ Density of the body = **6040.00 kg/m³. Ans.**

4.4. META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (*a*). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion

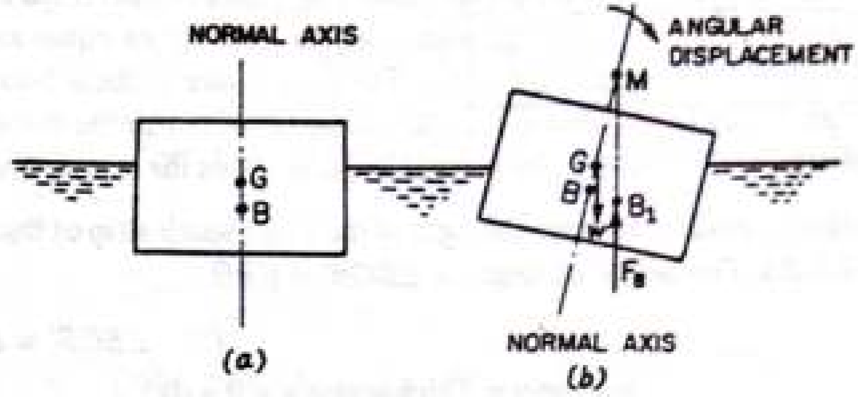


Fig. 4.5. Meta-centre.

of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

4.5. META-CENTRIC HEIGHT

The distance MG, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

4.6. ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 (*a*) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at G and B. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig. 4.6 (*b*). The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at M. Hence M is the meta-centre and GM is meta-centric height.

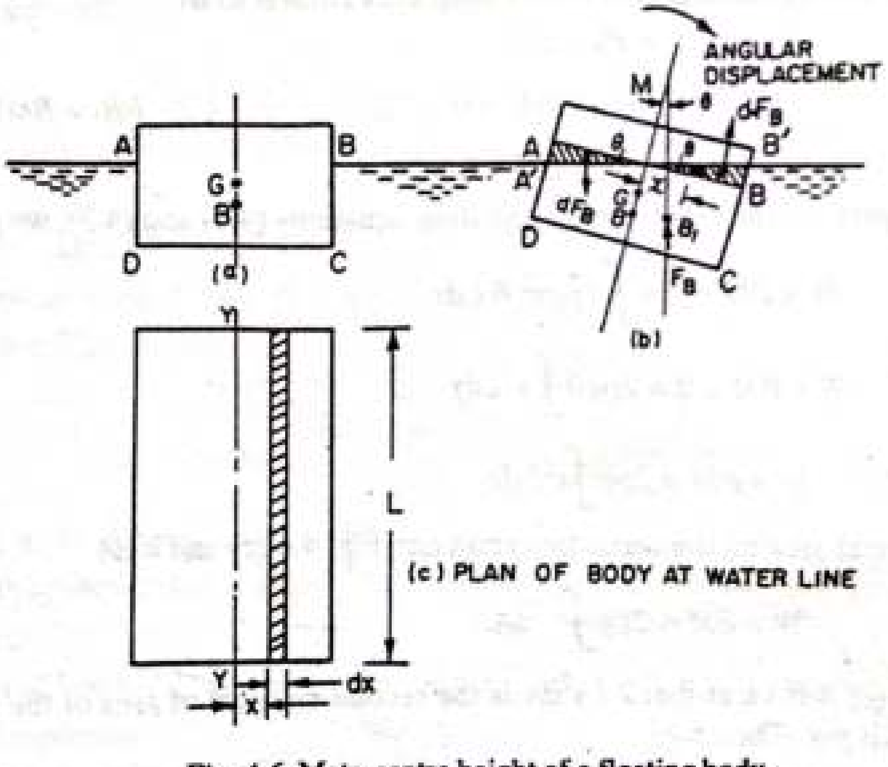


Fig. 4.6. Meta-centre height of a floating body.

The angular displacement of the body in the clockwise direction causes the wedge-shaped prism BOB' on the right of the axis to go inside the water while the identical wedge-shaped prism represented by AOA' emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the C.G. of the prism BOB' while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of AOA'. The couple due to these buoyant forces dF_B tends to rotate the ship in the counter clockwise direction. Also the moment caused by the displacement of the centre of buoyancy from B to B_1 is also in the counter clockwise direction. Thus these two couples must be equal.

Couple Due to Wedges. Consider towards the right of the axis a small strip of thickness dx at a distance x from O as shown in Fig. 4.5 (*b*). The height of strip $x \times \angle BOB' = x \times \theta$.

$\{\because \angle BOB' = \angle AOA' = BMB_1' = \theta\}$

$\therefore$ Area of strip $= \text{Height} \times \text{Thickness} = x \times \theta \times dx$

If L is the length of the floating body, then

Volume of strip $= \text{Area} \times L$

$= x \times \theta \times L \times dx$

$\therefore$ Weight of strip $= \rho g \times \text{Volume} = \rho g x\ \theta L\ dx$

Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of strip will be $wx\theta L\ dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple $=$ Weight of each strip $\times$ Distance between these two weights

$= \rho g x\ \theta L\ dx\ [x + x]$

$= \rho g x\ \theta L\ dx \times 2x = 2\rho g x^2\ \theta L\ dx$

$\therefore$ Moment of the couple for the whole wedge

$$= \int 2\rho g x^2\ \theta L\ dx \qquad \text{...(4.1)}$$

Moment of couple due to shifting of centre of buoyancy from B to B_1

$= F_B \times BB_1$

$= F_B \times BM \times \theta$ $\{\because BB_1 = BM\theta$ if θ is very small$\}$

$= W \times BM \times \theta$ $\{\because F_B = W\}$ ...(4.2)

But these two couples are the same. Hence equating equations (4.1) and (4.2), we get

$$W \times BM \times \theta = \int 2\rho g x^2\ \theta\ L dx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$$

$$W \times BM = 2\rho g \int x^2 L dx$$

Now Ldx = Elemental area on the water line shown in Fig. 4.6 (*c*) and $= dA$

$$\therefore \quad W \times BM = 2\rho g \int x^2 dA.$$

But from Fig. 4.5 (*c*) it is clear that $2 \int x^2 dA$ is the second moment of area of the plan of the body at water surface about the axis *y-y*. Therefore

$$W \times BM = \rho g I \qquad \{\text{where } I = 2 \int x^2 dA\}$$

$$\therefore \quad BM = \frac{\rho g I}{W}$$

But W = Weight of the body

= Weight of the fluid displaced by the body

= ρg × Volume of the fluid displaced by the body

= ρg × Volume of the body sub-merged in water

$= \rho g \times \forall$

$$\therefore \quad BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \quad \text{...(4.3)}$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

$$\therefore \quad \text{Meta-centric height } = GM = \frac{I}{\forall} - BG. \quad \text{...(4.4)}$$

Problem 4.7. *A rectangular pontoon is 5 m long, 3 m wide and 1.20 m hight. The depth of immersion of the pontoon is 0.80 m in sea water. If the centre of gravity is 0.6 m above the bottom of the pontoon, determine the meta-centric height. The density for sea water = 1025 kg/m³.* (Delhi University, 1992)

Sol. Given :

Dimension of pontoon $= 5 \text{ m} \times 3 \text{ m} \times 1.20 \text{ m}$

Depth of immersion $= 0.8$ m

Distance $AG = 0.6$ m

Distance $AB = \frac{1}{2} \times$ Depth of immersion

$= \frac{1}{2} \times .8 = 0.4$ m

Density for sea water $= 1025 \text{ kg/m}^3$

Meta-centre height is GM is given by Equation (4.4) is

$$GM = \frac{I}{\forall} - BG$$

where I = M.O. Inertia of the plan of the pontoon about y-y axis

$$= \frac{1}{12} \times 5 \times 3^3 \text{ m}^4 = \frac{45}{4} \text{ m}^4$$

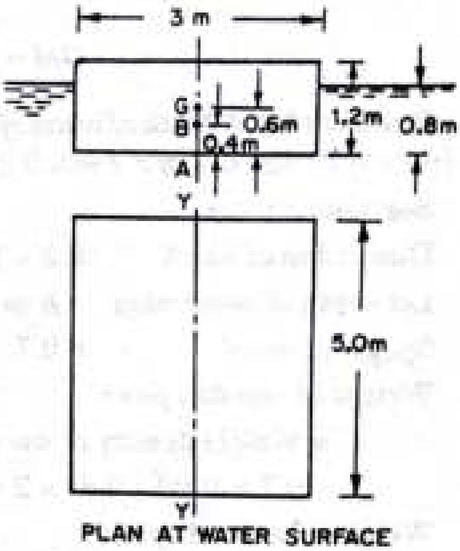


Fig. 4.7

$\forall$ = Volume of the body sub-merged in water

$= 3 \times 0.8 \times 5.0 = 12.0 \text{ m}^3$

$$BG = AG - AB = 0.6 - 0.4 = 0.2 \text{ m}$$

$$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = \mathbf{0.7375 \text{ m.}} \quad \textbf{Ans.}$$

Problem 4.8. *A uniform body of size 3 m long × 2 m wide × 1 m deep floats in water. What is the weight of the body if depth of immersion is 0.8 m ? Determine the meta-centric height also.*

Sol. Given :

Dimension of body $= 3 \times 2 \times 1$

Depth of immersion $= 0.8$ m

Find (*i*) Weight of body, W

(*ii*) Meta-centric height, GM

(*i*) **Weight of Body, W**

= Weight of water displaced = ρg × Volume of water displaced

$= 1000 \times 9.81 \times$ Volume of body in water

$= 1000 \times 9.81 \times 3 \times 2 \times 0.8$ N

$=$ **47088 N. Ans.**

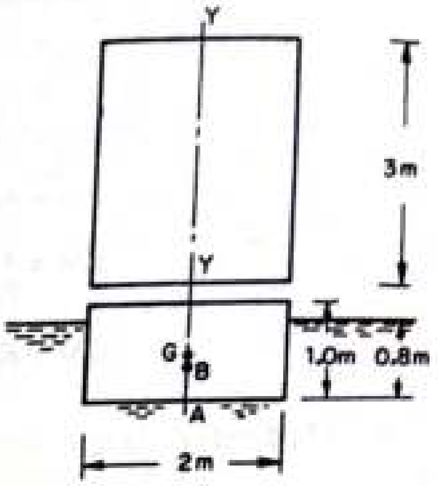


Fig. 4.8

(ii) Meta-centric Height, GM

Using equation (4.4), we get

$$GM = \frac{I}{\forall} - BG$$

where $I =$ M.O.I. about *y-y* axis of the plan of the body

$$= \frac{1}{12} \times 3 \times 2^3 = \frac{3 \times 2^3}{12} = 2.0 \text{ m}^4$$

$\forall =$ Volume of body in water

$= 3 \times 2 \times 0.8 = 4.8 \text{ m}^3$

$$BG = AG - AB = \frac{1.0}{2} - \frac{0.8}{2} = 0.5 - 0.4 = 0.1$$

$\therefore$
$$GM = \frac{2.0}{4.8} - 0.1 = 0.4167 - 0.1 = \mathbf{0.3167 \text{ m}}. \quad \textbf{Ans.}$$

Problem 4.9. *A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2 m × 1 m × 0.8 m.*

Sol. Given :

Dimension of block $= 2 \times 1 \times 0.8$

Let depth of immersion $= h$ *m*

Sp. gr. of wood $= 0.7$

Weight of wooden piece

$=$ Weight density of wood* × Volume

$= 0.7 \times 1000 \times 9.81 \times 2 \times 1 \times 0.8$ N

Weight of water displaced

$=$ Weight density of water

× Volume of the wood sub-merged in water

$= 1000 \times 9.81 \times 2 \times 1 \times h$ N

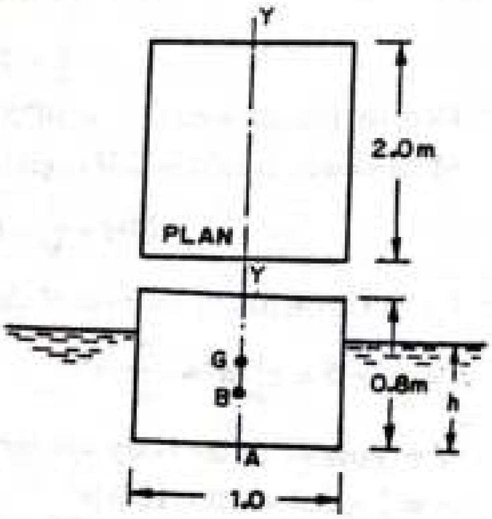


Fig. 4.9

For equilibrium,

Weight of wooden piece $=$ Weight of water displaced

$\therefore \quad 700 \times 9.81 \times 2 \times 1 \times 0.8 = 1000 \times 9.81 \times 2 \times 1 \times h$

$\therefore$
$$h = \frac{700 \times 9.81 \times 2 \times 1 \times 0.8}{1000 \times 9.81 \times 2 \times 1} = 0.7 \times 0.8 = 0.56 \text{ m}$$

$\therefore$ Distance of centre of Buoyancy from bottom, *i.e.*,

$$AB = \frac{h}{2} = \frac{0.56}{2} = 0.28 \text{ m}$$

and
$$AG = 0.8/2.0 = 0.4 \text{ m}$$

$\therefore$
$$BG = AG - AB = 0.4 - 0.28 = 0.12 \text{ m}$$

---

*Weight density of wood $= \rho \times g$, where $\rho =$ density of wood

$= 0.7 \times 1000 = 700 \text{ kg/m}^3$. Hence w for wood $= 700 \times 9.81 \text{ N/m}^3$.

The meta-centric height is given by equation (4.4) or

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{1}{12} \times 2 \times 1.0^3 = \frac{1}{6}\ \text{m}^4$

$\forall$ = Volume of wood in water

$= 2 \times 1 \times h = 2 \times 1 \times .56 = 1.12\ \text{m}^3$

$\therefore$ $$GM = \frac{1}{6} \times \frac{1}{1.12} - 0.12 = 0.1488 - 0.12 = \mathbf{0.0288\ m.} \quad \textbf{Ans.}$$

Problem 4.10. *A solid cylinder of diameter 4.0 m has a height of 3 metres. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The sp. gr. of the cylinder = 0.6.*

Sol. Given :

Dia. of cylinder, $D = 4.0$ m

Height of cylinder, $h = 3.0$ m

Sp. gr. of cylinder $= 0.6$

Depth of immersion of cylinder

$= 0.6 \times 3.0 = 1.8$ m

$\therefore$ $AB = \frac{1.8}{2} = 0.9$ m

and $AG = \frac{3}{2} = 1.5$ m

$\therefore$ $BG = AG - AB$

$= 1.5 - 0.9 = 0.6$ m

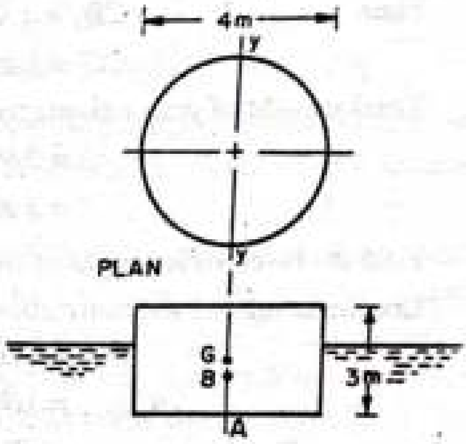


Fig. 4.10

Now the meta-centric height GM is given by equation (4.4)

$$GM = \frac{I}{\forall} - BG$$

But I = M.O.I. about y-y axis of the plan of the body

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

and $\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4} D^2 \times \text{Depth of immersion}$$

$$= \frac{\pi}{4} (4)^2 \times 1.8\ \text{m}^3$$

$\therefore$ $$GM = \frac{\frac{\pi}{64} \times (4.0)^4}{\frac{\pi}{4} \times (4.0)^2 \times 1.8} - 0.6$$

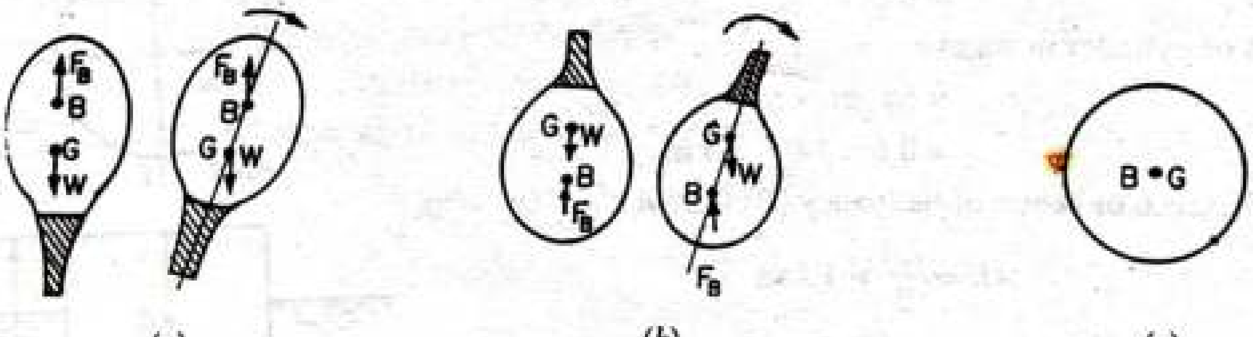
$$= \frac{1}{16} \times \frac{4.0^2}{1.8} - 0.6 = \frac{1}{1.8} - 0.6 = 0.55 - 0.6 = \mathbf{-0.05\ m.} \quad \textbf{Ans.}$$

– ve sign means that meta-centre, (M) is below the centre of gravity (G).

4.7. CONDITIONS OF EQUILIBRIUM OF A FLOATING AND SUB-MERGED BODIES

A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of the centre of gravity (G) and centre of buoyancy (B_1) of a body determines the stability of a sub-merged body.

4.7.1. Stability of a Sub-merged Body. The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed. Consider a balloon, which is completely sub-merged in air. Let the lower portion of the balloon contains heavier material, so that its centre of gravity is lower than its centre of buoyancy as shown in Fig. 4.12 (*a*). Let the weight of the balloon is W. The weight W is acting through G, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through B. For the equilibrium of the balloon $W = F_B$. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. 4.12 (*a*), then W and F_B constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by Fig. 4.12 (*a*) is in stable equilibrium.



(*a*) STABLE EQUILIBRIUM (*b*) UNSTABLE EQUILIBRIUM (*c*) NEUTRAL EQUILIBRIUM

Fig. 4.12. Stabilities of sub-merged bodies.

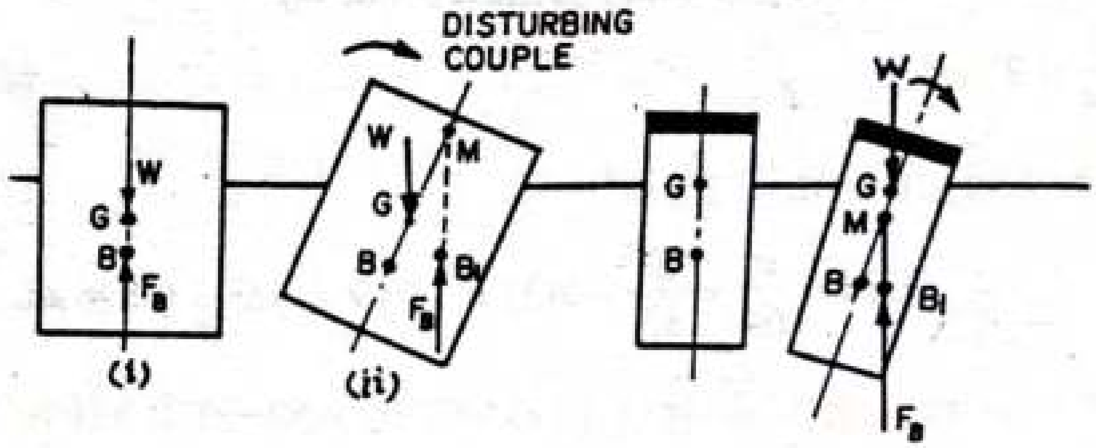
(*a*) **Stable Equilibrium.** When $W = F_B$ and point B is above G, the body is said to be in stable equilibrium.

(*b*) **Unstable Equilibrium.** If $W = F_B$, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium as shown in Fig. 4.12 (*b*). A slight displacement to the body, in the clockwise direction, gives the couple due to W and F_B also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(*c*) **Neutral Equilibrium.** If $F_B = W$ and B and G are at the same point, as shown in Fig. 4.12 (*c*), the body is said to be in Neutral Equilibrium.

4.7.2. Stability of Floating Body. The stability of a floating body is determined from the position of Meta-centre (*M*). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(*a*) **Stable Equilibrium.** If the point *M* is above *G*, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from *B* to B_1 such that the vertical line through B_1 cuts at *M*. Then the buoyant force F_B through B_1 and weight *W* through *G* constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.



(*a*) Stable equilibrium *M* is above *G* (*b*) Unstable equilibrium *M* is below *G*.

Fig. 4.13. Stability of floating bodies.

(*b*) **Unstable Equilibrium.** If the point *M* is below *G*, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and *W* is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point *M* is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

A. KINEMATICS OF FLOW

5.1. INTRODUCTION

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined. In this chapter, the methods of determining velocity and acceleration are discussed.

5.2. METHODS OF DESCRIBING FLUID MOTION

The fluid motion is described by two methods. They are— (*i*) **Lagrangian Method**, and (*ii*) **Eulerian Method**. In the Lagrangian method, a **single fluid particle** is followed during its motion and its velocity, acceleration, density, etc. are described. In case of Eulerian method. the velocity, acceleration, pressure, density etc. are described **at a point** in flow field. The Eulerian method is commonly used in fluid mechanics.

5.3. TYPES OF FLUID FLOW

The fluid flow is classified as :

(*i*) Steady and unsteady flows ;

(*ii*) Uniform and non-uniform flows ;

(*iii*) Laminar and turbulent flows ;

(*iv*) Compressible and incompressible flows ;

(*v*) Rotational and irrotational flows ; and

(*vi*) One, two and three-dimensional flows.

5.3.1. Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure of density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

5.3.2. Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity

∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

5.3.3. Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

5.3.4. Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

5.3.5. Rotational and Irrotational Flows. Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis that type of flow is called irrotational flow.

5.3.6. One, Two and Three-Dimensional Flows

One-dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two dimensional flow

$$u = f_1(x, y), \quad v = f_2(x, y) \quad \text{and} \quad w = 0.$$

Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z), \quad v = f_2(x, y, z), \quad w = f_3(x, y, z).$$

5.4. RATE OF FLOW OR DISCHARGE (Q)

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(*i*) For liquids the units of Q are m^3/s or litres/s

(*ii*) For gases the units of Q is kgf/s or Newton/s

Consider a liquid flowing through a pipe in which

A = Cross-sectional area of pipe

V = Average velocity of fluid across the section

Then discharge $\quad Q = A \times V.$ ...(5.1)

5.5. CONTINUITY EQUATION

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in Fig. 5.1.

Let $\quad V_1$ = Average velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

and $\quad V_2, \rho_2, A_2$ are corresponding valves at section, 2-2.

Then rate of flow at section 1-1 $= \rho_1 A_1 V_1$

Rate of flow at section 2-2 $= \rho_2 A_2 V_2$

According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

or $$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(5.2)$$

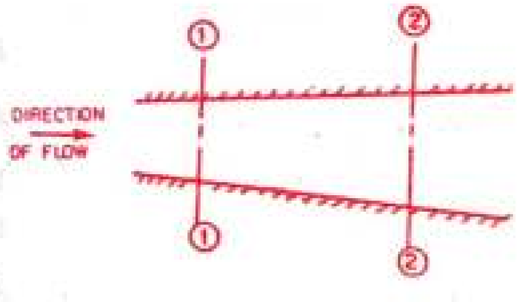


Fig. 5.1. Fluid flowing through a pipe.

Equation (5.2) is applicable to the compressible as well as incompressible fluids and is called **Continuity Equation.** If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (5.2) reduces to

$$A_1 V_1 = A_2 V_2 \quad ...(5.3)$$

Problem 5.1. *The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.*

Sol. Given :

At section 1, $\quad D_1 = 10 \text{ cm} = 0.1 \text{ m}.$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = .007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2, $D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

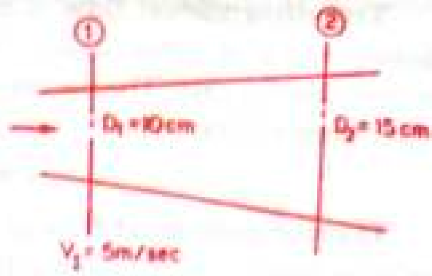


Fig. 5.2

(*i*) Discharge through pipe is given by equation (5.1)

or

$$Q = A_1 \times V_1$$

$$= .007854 \times 5 = 0.03927 \text{ m}^3/\text{s.} \quad \textbf{Ans.}$$

Using equation (5.3), we have $A_1V_1 = A_2V_2$

(*ii*) $\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.007854}{.01767} \times 5.0 = 2.22 \text{ m/s.} \quad \textbf{Ans.}$

Problem 5.2. *A 30 cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.*

Sol. Given :

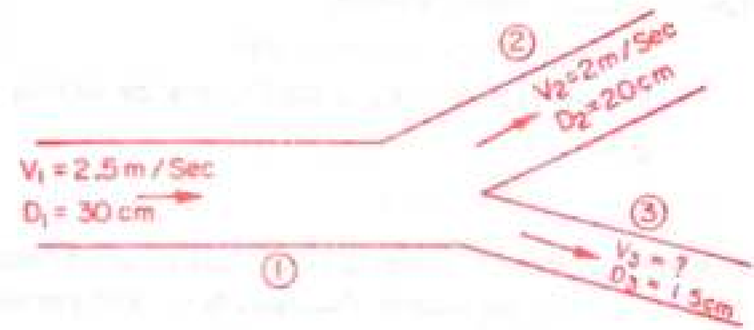


Fig. 5.3

$$D_1 = 30 \text{ cm} = 0.30 \text{ m}$$

$$\therefore \quad A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$\therefore \quad A_2 = \frac{\pi}{4}(.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$$

$$V_2 = 2 \text{ m/s}$$

$$D_3 = 15 \text{ cm} = 0.15 \text{ m}$$

$$\therefore \quad A_3 = \frac{\pi}{4}(.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$$

Find **(*i*) Discharge in pipe 1 or Q_1**

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3 \quad \text{...(1)}$$

(*i*) The discharge Q_1 in pipe 1 is given by

$$Q_1 = A_1V_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767 \text{ m}^3/\text{s}}. \quad \textbf{Ans.}$$

(*ii*) Value of V_3

$$Q_2 = A_2V_2 = .0314 \times 2.0 = .0628 \text{ m}^3/\text{s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

$\therefore$ $$Q_3 = .1767 - 0.0628 = 0.1139 \text{ m}^3/\text{s}$$

But $$Q_3 = A_3 \times V_3 = .01767 \times V_3 \quad \text{or} \quad .1139 = .01767 \times V_3$$

$\therefore$ $$V_3 = \frac{.1139}{.01767} = \mathbf{6.44 \text{ m/s}}. \quad \textbf{Ans.}$$

Dynamics of Fluid Flow

6.1. INTRODUCTION

In the previous chapter, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration the forces causing the flow. This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

6.2. EQUATIONS OF MOTION

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m.a_x \quad ...(6.1)$$

In the fluid flow, the following forces are present :

(i) F_g, gravity force.

(ii) F_p, the pressure force.

(iii) F_v, force due to viscosity.

(iv) F_t, force due to turbulence.

(v) F_c, force due to compressibility.

Thus in equation (6.1), the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(i) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called **Reynold's equations of motion.**

(ii) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(iii) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

6.3. EULER'S EQUATION OF MOTION

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in S-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are :

1. Pressure force pdA in the direction of flow.

2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.

3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element × acceleration in the direction S.

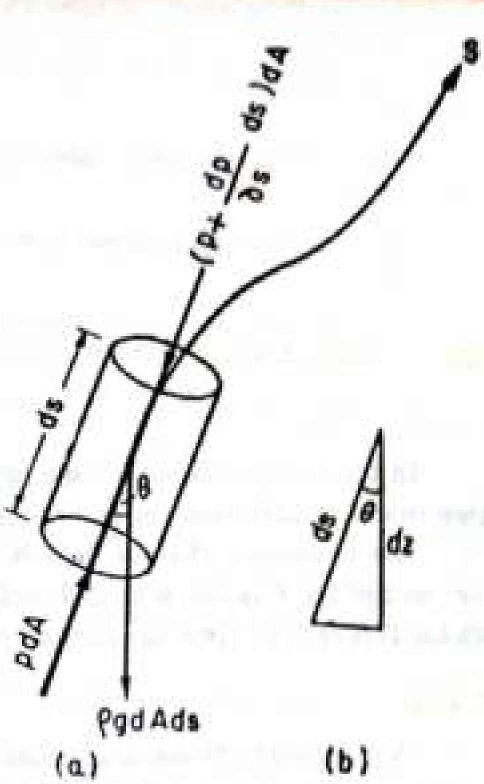


Fig. 6.1. Forces on a fluid element.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s \quad ...(6.2)$$

where a_s is the acceleration in the direction of S.

Now $a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \qquad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho\, dA ds \times \frac{v\partial v}{\partial s}$$

Dividing by $\rho ds dA$,

$$-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

or

$$\frac{\partial p}{\rho \partial s} + g\cos\theta + \frac{v\partial v}{\partial s} = 0$$

But from Fig. 6.1 (*b*), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \qquad \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0 \quad \text{or} \quad \frac{\partial p}{\rho} + gdz + vdv = 0$$

or

$$\frac{\partial p}{\rho} + gdz + vdv = 0 \qquad ...(6.3)$$

Equation (6.3) is known as Euler's equation of motion.

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int gdz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or $$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or $$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \quad ...(6.4)$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ **= pressure energy per unit weight of fluid or pressure Head.**

$v^2/2g$ **= kinetic energy per unit weight or kinetic Head.**

z **= potential energy per unit weight or potential Head.**

6.5. ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.* viscosity is zero

(*ii*) The flow is steady

(*iii*) The flow is incompressible

(*iv*) The flow is irrotational.

Sol. Statement of Bernoulli's Theorem. It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are :

Pressure energy $= \frac{p}{\rho g}$

Kinetic energy $= \frac{v^2}{2g}$

Datum energy $= z$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{w} + \frac{v^2}{2g} + z = \text{Constant.}$$

Derivation of Bernoulli's theorem. For derivation of Bernoulli's theorem, the Articles 6.3 and 6.4 should be written.

Assumptions are given in Article 6.5.

Problem 6.4. *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

Sol. Given :

At Section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$A_1 = \frac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

$p_1 = 39.24 \text{ N/cm}^2$

$= 39.24 \times 10^4 \text{ N/m}^2$

$z_1 = 6.0 \text{ m}$

At Section 2, $D_2 = 0.10 \text{ m}$

$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$

$z_2 = 4 \text{ m}$

$p_2 = ?$

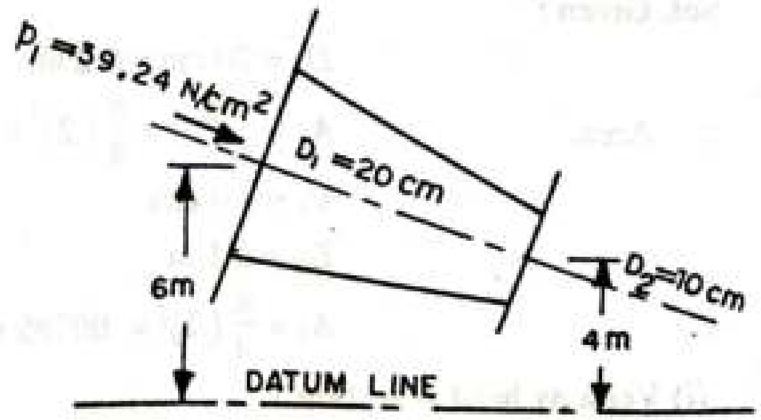


Fig. 6.3

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = 0.035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \frac{Q}{A_1} = \frac{0.035}{.0314} = 1.114 \text{ m/s}$

and $V_2 = \frac{Q}{A_2} = \frac{0.035}{.00785} = 4.456 \text{ m/s}$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}. \quad \textbf{Ans.}$$

Problem 6.5. *Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.*

Sol. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

or $Q = \frac{40}{1000} = 0.04 \text{ m}^3\text{/s}$

Now $A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$

$\therefore$ $$V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(.2)^2} = 1.274 \text{ m/s}$$

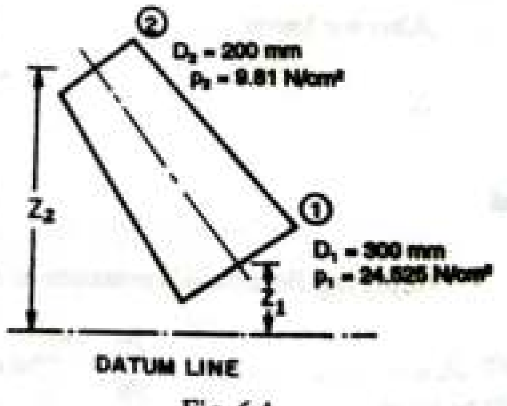


Fig. 6.4

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

or $$\frac{24.525\times10^4}{1000\times9.81}+\frac{.566\times.566}{2\times9.81}+z_1=\frac{9.81\times10^4}{1000\times9.81}+\frac{(1.274)^2}{2\times9.81}+z_2$$

or $$25+.32+z_1=10+1.623+z_2$$

or $$25.32+z_1=11.623+z_2$$

$\therefore$ $$z_2-z_1=25.32-11.623=13.697=13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem 6.6. *The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².*

Sol. Given :

Length of pipe, $L = 100$ m

Dia. at the upper end, $D_1 = 600$ mm $= 0.6$ m

$\therefore$ Area, $A_1=\frac{\pi}{4}D_1^2=\frac{\pi}{4}\times(.6)^2=0.2827 \text{ m}^2$

p_1 = pressure at upper end = 19.62 N/cm² $= 19.62\times10^4$ N/m²

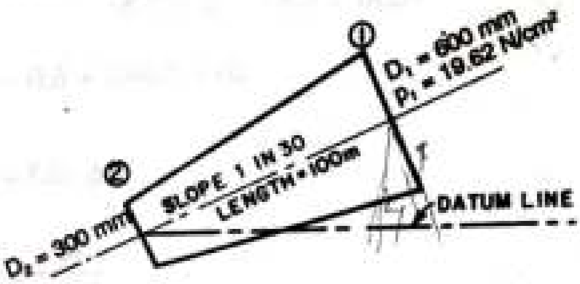


Fig. 6.5

Dia. at lower end, $D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_2=\frac{\pi}{4}D_2^2=\frac{\pi}{4}(.3)^2=0.07068 \text{ m}$

Q = rate of flow = 50 litres/s $=\frac{50}{1000}=0.05 \text{ m}^3/\text{s}$

Let the datum line is passing through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1=\frac{1}{30}\times100=\frac{10}{3}$ m

Also we know $Q = A_1V_1 = A_2V_2$

$\therefore$ $$V_1=\frac{Q}{A}=\frac{0.05}{.2827}=0.1768 \text{ m/sec} = 0.177 \text{ m/s}$$

and $$V_2=\frac{Q}{A_2}=\frac{0.5}{.07068}=0.7074 \text{ m/sec}=0.707 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

or $$\frac{19.62\times10^4}{1000\times9.81}+\frac{.177^2}{2\times9.81}+\frac{10}{3}=\frac{p_2}{\rho g}+\frac{.707^2}{2\times9.81}+0$$

or $$20+.001596+3.334=\frac{p_2}{\rho g}+0.0254$$

or $$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or $$p_2 = 23.3 \times 9810 \text{ N/m}^2 = 228573 \text{ N/m}^2 = \mathbf{22.857 \text{ N/cm}^2}.$$

6.6. BERNOULLI'S EQUATION FOR REAL FLUID

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between point 1 and 2 is given as

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \qquad \text{...(6.5)}$$

where h_L is loss of energy between point 1 and 2.

6.7. PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1. Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for Rate of Flow Through Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

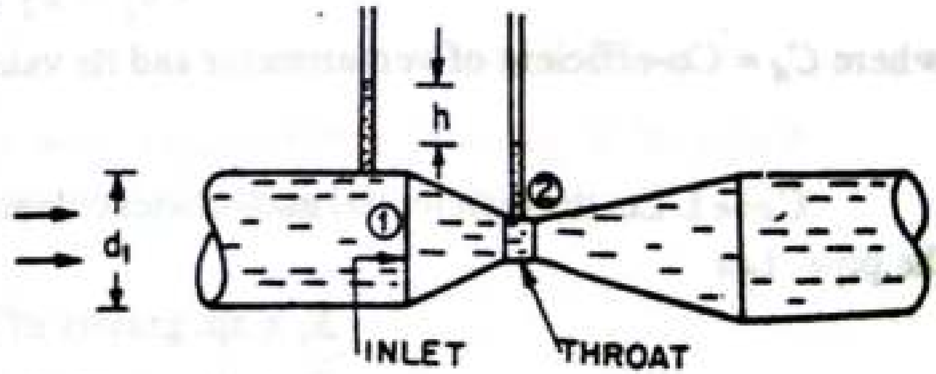


Fig. 6.9. Venturimeter.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \quad v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge, $Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \quad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = sp. gravity of the heavier liquid

S_o = sp. gravity of the liquid flowing through pipe

x = difference of the heavier liquid column in U-tube

Then
$$h = x\left[\frac{S_h}{S_o} - 1\right] \qquad \text{...(6.9)}$$

Case II. If the differential manometer contains a liquid which lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \quad ...(6.10)$$

where S_l = sp. gr. of lighter liquid in U-tube

S_o = sp. gr. of fluid flowing through pipe

x = difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \quad ...(6.11)$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \quad ...(6.12)$$

Problem 6.10. *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take C_d = 0.98.*

Sol. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85\ \text{cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7\ \text{cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

∴ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = sp. gravity of mercury = 13.6, S_o = sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by Eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s.}} \quad \textbf{Ans.}$$

Problem 6.11. *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Sol. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

∴ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

∴ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

∴ The discharge Q is given by equation (6.8)

or $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s.}} \quad \textbf{Ans.}$$

Problem 6.12. *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 litres/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$.*

Sol. Given : $d_1 = 20$ cm

∴ $a = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$

Using the equation (6.8), $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78 \sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore$ $$h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

where
S_h = sp. gr. of mercury = 13.6
S_o = sp. gr. of oil = 0.8
x = Reading of manometer

$\therefore$ $$289.98 = x \left[\frac{13.6}{0.8} - 1 \right] = 16x$$

$\therefore$ $$x = \frac{289.98}{16} = 18.12 \text{ cm.} \quad \textbf{Ans.}$$

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

Problem 6.13. *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm² and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. Take C_d = 0.98.*

Sol. Given :

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$

$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

p for water $= 1000 \frac{\text{kg}}{\text{m}^3}$ and $\therefore \frac{p_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$$\frac{p_2}{\rho g} = -30 \text{ cm of mercury}$$
$$= -0.30 \text{ m of mercury} = -0.30 \times 13.6 = -4.08 \text{ m of water}$$

$\therefore$ Differential head $$= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$$
$$= 18 + 4.08 = 22.08 \text{ m of water} = 2208 \text{ cm of water}$$

The discharge Q is given by equation (6.8)

$\therefore$ $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$
$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$
$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3\text{/s} = \textbf{165.555 lit/s.} \quad \textbf{Ans.}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3/\text{s} = \textbf{137.414 litres/s.} \quad \textbf{Ans.}$$

6.7.3. Pitot-tube. It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig. 6.13.

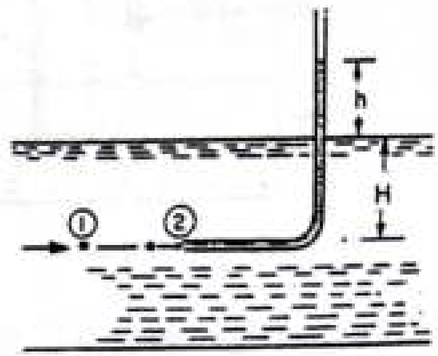


Fig. 6.13. Pitot-tube.

The lower end, which is bent through 90° is directed in the upstream direction as shown in Fig. 6.13. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just at the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equations at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H)$$

$$\therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $v = C_v \sqrt{2gh}$ ...(6.14)

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig. 6.14.
2. Pitot-tube connected with piezometer tube as shown in Fig. 6.15.

3. Pitot-tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig. 6.16.

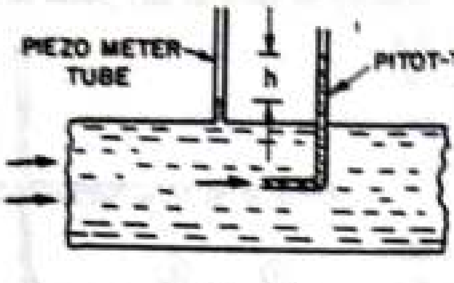


Fig. 6.14

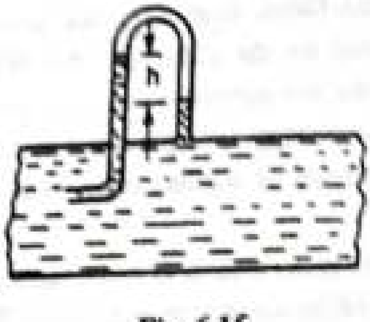


Fig. 6.15

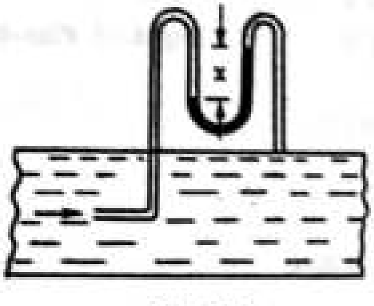


Fig. 6.16

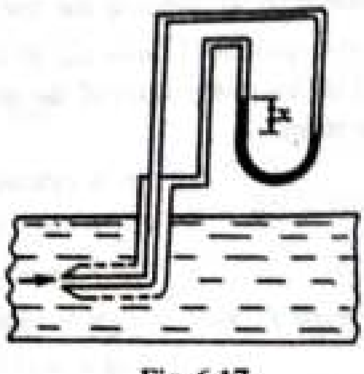


Fig. 6.17

4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. 6.17. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_o} - 1\right]$.

Problem 6.24. *A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as $C_v = 0.98$.*

Sol. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Diff. of pressure head, $h = 60$ mm of water $= .06$ m of water

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80 \times$ Central velocity

Central velocity is given by equation (6.14)

$$= C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore$ $\overline{V} = 0.80 \times 1.063 = 0.8504$ m/s

Discharge, $Q = \text{Area of pipe} \times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4}(.30)^2 \times 0.8504 = \mathbf{0.06 \ m^3/s.} \quad \textbf{Ans.}$$

Problem 6.25. *Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.*

Sol. Given :

Diff. of mercury level, $x = 100 \text{ mm} = 0.1 \text{ m}$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

$C_v = 0.98$

Diff. of pressure head, $h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6 \text{ m of oil}$

$\therefore$ Velocity of flow $= C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6} = \mathbf{5.49 \text{ m/s}}$. **Ans.**

Problem 6.26. *A pitot-static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.* (A.M.I.E., Winter, 1979)

Sol. Given :

Stagnation pressure head, $h_s = 6 \text{ m}$

Static pressure head, $h_t = 5 \text{ m}$

$\therefore$ $h = 6 - 5 = 1 \text{ m}$

Velocity of flow, $V = C_v\sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1} = \mathbf{4.34 \text{ m/s}}$. **Ans.**

Problem 6.27. *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.* (A.M.I.E., Winter, 1975)

Sol. Given :

Diff. of mercury level, $x = 170 \text{ mm} = 0.17 \text{ m}$

Sp. gr. of mercury, $S_g = 13.6$

Sp. gr. of sea-water, $S_o = 1.026$

$\therefore$ $h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834 \text{ m}$

$\therefore$ $V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393 \text{ m/s}$

$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01 \text{ km/hr}}$. **Ans.**

Problem 6.28. *A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, recorded by the pitot-tube is 0.981 N/cm². Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$.* (Converted to S.I. Units, A.M.I.E., Summer, 1987)

Sol. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(.3)^2 = 0.07068 \text{ m}^2$

Static pressure head $= 100$ mm of mercury (vacuum)

$$= -\frac{100}{1000} \times 13.6 = -1.36 \text{ m of water}$$

Stagnation pressure $= .981 \text{ N/cm}^2 = .981 \times 10^4 \text{ N/m}^2$

$\therefore$ Stagnation pressure head $= \frac{.981 \times 10^4}{\rho g} = \frac{.981 \times 10^4}{1000 \times 9.81} = 1 \text{ m}$

$\therefore$ h = Stagnation pressure head – Static pressure head

$$= 1.0 - (-1.36) = 1.0 + 1.36 = 2.36 \text{ m of water}$$

$\therefore$ Velocity at centre $= C_v \sqrt{2gh}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668 \text{ m/s}$$

Mean velocity, $\overline{V} = 0.85 \times 6.668 = 5.6678$ m/s

$\therefore$ Rate of flow of water $= \overline{V} \times$ area of pipe

$= 5.6678 \times .07068 \text{ m}^3\text{/s} =$ **$0.4006 \text{ m}^3\text{/s}$.** **Ans.**

7.1. INTRODUCTION

Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank or vessel containing the fluid. Orifices as well as mouthpieces are used for measuring the rate of flow of fluid.

7.2. CLASSIFICATIONS OF ORIFICES

The orifices are classified on the basis of their size, shape, nature of discharge and shape of the upstream edge. The followings are the important classifications :

1. The orifices are classified as **small orifice** or **large orifice** depending upon the size of orifice and head of liquid from the centre of the orifice. If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice. And if the head of liquids is less than five times the depth of orifice, it is known as large orifice.

2. The orifices are classified as (*i*) Circular orifice, (*ii*) Triangular orifice, (*iii*) Rectangular orifice and (*iv*) Square orifice depending upon their cross- sectional areas.

3. The orifices are classified as (*i*) Sharp-edged orifice and (*ii*) Bell-mouthed orifice depending upon the shape of upstream edge of the orifices.

4. The orifices are classified as (*i*) Free discharging orifices and (*ii*) Drowned or Sub-merged orifices depending upon the nature of discharge.

The sub-merged orifices are further classified as (*a*) Fully sub-merged orifices and (*b*) Partially sub-merged orifices.

7.3. FLOW THROUGH ON ORIFICE

Consider a tank fitted with a circular orifice in one of its sides as shown in Fig. 7.1. Let H be the head of the liquid above the centre of the orifice. The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice. The area of jet of fluid goes on decreasing and at a section CC, the area is minimum. This section is approximately at a distance of half of diameter of the orifice. At this section, the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. This section is called **Vena-contracta.** Beyond this section, the jet diverges and is attracted in the downward direction by the gravity.

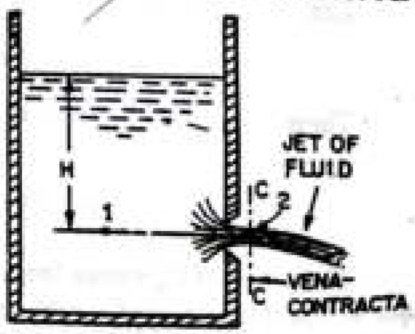


Fig. 7.1. Tank with an orifice.

Consider two points 1 and 2 as shown in Fig. 7.1. Point 1 is inside the tank and point 2 at the vena-contracta. Let the flow is steady and at a constant head H. Applying Bernoulli's equation at points 1 and 2.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But

$$z_1 = z_2$$

∴

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now

$$\frac{p_1}{\rho g} = H$$

$$\frac{p_2}{\rho g} = 0 \qquad \text{(atmospheric pressure)}$$

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

∴

$$H + 0 = 0 + \frac{v_2^2}{2g}$$

∴

$$v_2 = \sqrt{2gH} \qquad ...(7.1)$$

This is theoretical velocity. Actual velocity will be less than this value.

7.4. HYDRAULIC CO-EFFICIENTS

The hydraulic co-efficients are

1. Co-efficient of velocity, C_v
2. Co-efficient of contraction, C_c
3. Co-efficient of discharge, C_d.

7.4.1. Co-efficient of Velocity (C_v). It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by C_v and mathematically, C_v is given as

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}, \quad \text{where } V = \text{actual velocity, } \sqrt{2gH} = \text{Theoretical velocity.} \qquad ...(7.2)$$

The value of C_v varies from 0.95 to 0.99 for different orifices, depending on the shape, size of the orifice and on the head under which flow takes place. Generally the value of $C_v = 0.98$ is taken for sharp-edged orifices.

7.4.2. Co-efficient of Contraction (C_c). It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice. It is denoted by C_c.

Let

$$a = \text{area of orifice and}$$

$$a_c = \text{area of jet at vena-contracta.}$$

Then

$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$

$$= \frac{a_c}{a} \qquad ...(7.3)$$

The value of C_c varies from 0.61 to 0.69 depending on shape and size of the orifice and head of liquid under which flow takes place. In general, the value of C_c may be taken 0.64.

7.4.3. Co-efficient of Discharge (C_d). It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d. If Q is actual discharge and Q_{th} is the theoretical discharge then mathematically, C_d is given as

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$\therefore \quad C_d = C_v \times C_c \qquad ...(7.4)$$

The value of C_d varies from 0.61 to 0.65. For general purpose the value of C_d is taken as 0.62.

Problem 7.1. *The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.*

Sol. Given :

Head, $H = 10$ cm

Dia. of orifice, $d = 40$ mm $= 0.04$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.04)^2 = .001256 \text{ m}^2$

$C_d = 0.6$

$C_v = 0.98$

(*i*) $\frac{\text{Actual Discharge}}{\text{Theoretical Discharge}} = 0.6$

But Theoretical Discharge = V_{th} × Area of orifice

V_{th} = Theoretical velocity, where $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14$ m/s

$\therefore$ Theoretical Discharge $= 14 \times .001256 = 0.01758 \frac{\text{m}^2}{\text{s}}$

$\therefore$ Actual Discharge = 0.6 × Theoretical Discharge

$= 0.6 \times .01758 =$ **0.01054 m³/s. Ans.**

(*ii*) $\frac{\text{Actual Velocity}}{\text{Theoretical velocity}} = C_v = 0.98$

$\therefore$ Actual velocity = 0.98 × Theoretical velocity

$= 0.98 \times 14 =$ **13.72 m/s. Ans.**

Problem 7.2. *The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litre/s. Find the co-efficient of discharge.*

Sol. Given :

Dia. of orifice, $d = 20$ mm $= .02$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.02)^2 = .000314 \text{ m}^2$

Head, $H = 1$ m

Actual discharge, $Q = 0.85$ litres/s $= .00085 \text{ m}^3/\text{s}$

Theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429$ m/s

$\therefore$ Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice

$= 4.429 \times .000314 = 0.00139 \text{ m}^3/\text{s}$

$\therefore$ Co-efficient of discharge $= \frac{\text{Actual Discharge}}{\text{Theoretical Discharge}} = \frac{0.00085}{0.00139} =$ **0.61. Ans.**

Notches and Weirs

8.1. INTRODUCTION

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called **Nappe** or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

8.2. CLASSIFICATION OF NOTCHES AND WEIRS

The notches are classified as :

1. According to the shape of the opening :

(*a*) Rectangular notch,

(*b*) Triangular notch,

(*c*) Trapezoidal notch, and

(*d*) Stepped notch.

2. According to the effect of the sides on the nappe :

(*a*) Notch with end contraction.

(*b*) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :

(*i*) Rectangular weir, (*ii*) Triangular weir, and

(*iii*) Trapezoidal weir (Cippoletti weir)

(*b*) According to the shape of the crest :

(*i*) Sharp-crested weir, (*ii*) Broad-crested weir,

(*iii*) Narrow-crested weir, and (*iv*) Ogee-shaped weir.

(*c*) According to the effect of sides on the emerging nappe :

(*i*) Weir with end contraction, and (*ii*) Weir without end contraction.

8.3. DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The expression for discharge over a rectangular notch or weir is the same.

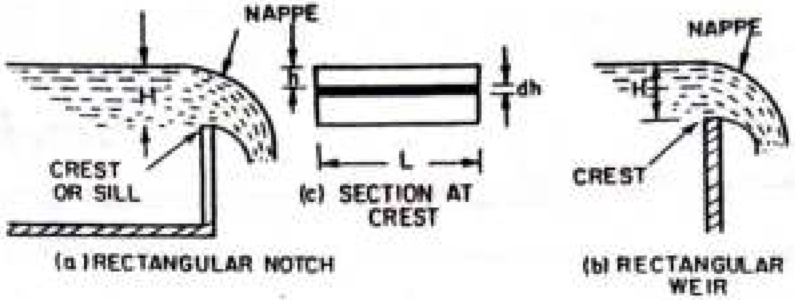


Fig. 8.1. Rectangular notch and weir.

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in Fig. 8.1 (*c*).

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \quad \text{...(i)}$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (*i*) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \,.\, L \,.\, \sqrt{2gh} \,.\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\, [H]^{3/2}. \quad \text{...(8.1)}$$

Problem 8.1. *Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.*

Sol. Given :

Length of the notch, $L = 2.0$ m

Head over notch, $H = 300$ m $= 0.30$ m

$C_d = 0.60$

Discharge, $Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\, [H^{3/2}]$

$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times [.30]^{1.5}$ m³/s

$= 3.5435 \times 0.1643 =$ **0.582 m³/s. Ans.**

Problem 8.2. *Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take $C_d = 0.6$ and neglect end contractions.*

Sol. Given :

Length of weir, $L = 6$ m

Depth of water, $H_1 = 1.8$ m

Discharge, $Q = 2000$ lit/s $= 2$ m^3/s

$C_d = 0.6$

Let H is height of water above the crest of weir, and H_2 = height of weir (Fig. 8.2)

The discharge over the weir is given by the equation (8.1) as

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\, H^{3/2}$$

or $$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623\, H^{3/2}$$

$\therefore$ $$H^{3/2} = \frac{2.0}{10.623}$$

$\therefore$ $$H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

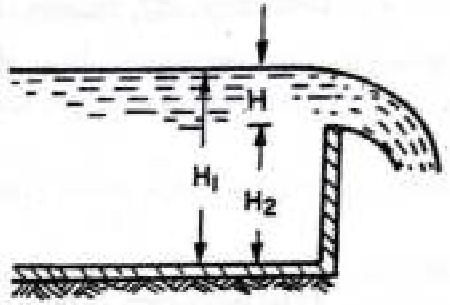


Fig. 8.2

$\therefore$ Height of weir, $H_2 = H_1 - H$

= Depth of water on upstream side $- H$

$= 1.8 - .328 =$ **1.472 m. Ans.**

Problem 8.3. *The head of water over a rectangular notch is 900 mm. The discharge is 300 litres/s. Find the length of the notch when $C_d = 0.62$.*

Sol. Given :

Head over notch, $H = 90$ cm $= 0.9$ m

Discharge, $Q = 300$ lit/s $= 0.3$ m^3/s

$C_d = 0.62$

Let length of notch $= L$

Using equation (8.1), we have

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or $$0.3 = \frac{2}{3} \times 0.62 \times L \times \sqrt{2 \times 9.81} \times (0.9)^{3/2}$$

$$= 1.83 \times L \times 0.8538$$

$\therefore$ $$L = \frac{0.3}{1.83 \times .8538} = .192 \text{ m} = \textbf{192 mm.} \quad \textbf{Ans.}$$

8.4. DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V-notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (*b*), we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$\therefore$ $$AC = (H-h)\tan\frac{\theta}{2}$$

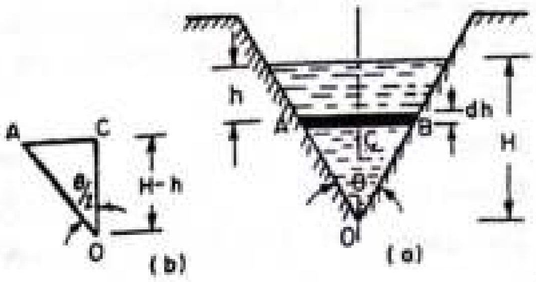


Fig. 8.3. The triangular notch.

Width of strip $= AB = 2AC = 2(H-h)\tan\frac{\theta}{2}$

$\therefore$ Area of strip $= 2(H-h)\tan\frac{\theta}{2} \times dh$

The theoretical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge, dQ, through the strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge, Q is $$Q = \int_0^H 2\,C_d(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \qquad \text{...(8.2)}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \qquad \therefore \tan\frac{\theta}{2} = 1$$

Discharge $$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$= 1.417\,H^{5/2}. \qquad \text{...(8.3)}$$

Problem 8.4. *Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$.*

Sol. Given :

Angle of *V*-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.6$

Discharge, Q over a *V*-notch is given by equation (8.2)

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \tan\frac{60}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.8182 \times 0.0493 = \mathbf{0.040\ m^3/s.}\quad \textbf{Ans.}$$

Problem 8.5. *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

(Osmania University, 1990 ; A.M.I.E., Winter, 1975)

Sol. Given :

For rectangular weir, Length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir, $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

The discharge over the rectangular weir is given by equation (8.1) as

$$Q = \tfrac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \tfrac{2}{3} \times 0.62 \times 1.0 \times \sqrt{2 \times 9.81} \times (.15)^{3/2}\ \text{m}^3/\text{s} = 0.10635\ \text{m}^3/\text{s}$$

The same discharge passes through the triangular right-angled weir. But discharge, Q, is given by equation (8.2) for a triangular weir as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad 0.10635 = \frac{8}{15} \times .59 \times \tan\frac{90}{2} \times \sqrt{2g} \times {H_1}^{5/2} \qquad \{\because\ \theta = 90° \text{ and } H = H_1\}$$

$$= \frac{8}{15} \times .59 \times 1 \times 4.429 \times {H_1}^{5/2} = 1.3936\, {H_1}^{5/2}$$

$$\therefore \quad {H_1}^{5/2} = \frac{0.10635}{1.3936} = 0.07631$$

$$\therefore \quad H_1 = (.07631)^{0.4} = \mathbf{0.3572\ m.}\quad \textbf{Ans.}$$

Problem 8.5A. *Water flows through a triangular right-angled weir first and then over a rectangular weir of 1 m width. The discharge co-efficients of the triangular and rectangular weirs are 0.6 and 0.7 respectively. If the depth of water over the triangular weir is 360 mm, find the depth of water over the rectangular weir.*

(A.M.I.E., Summer, 1990)

Sol. Given :

For triangular weir : $\theta = 90°$, $C_d = 0.6$, $H = 360$ mm $= 0.36$ m

For rectangular weir : $L = 1$ m, $C_d = 0.7$, $H = ?$

The discharge for a triangular weir is given by equation (8.2) as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan\left(\frac{90}{2}\right) \times \sqrt{2 \times 9.81} \times (0.36)^{5/2} = 0.1102 \text{ m}^3\text{/s}$$

The same discharge is passing through the rectangular weir. But discharge for a rectangular weir is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

or
$$0.1102 = \frac{2}{3} \times 0.7 \times 1 \times \sqrt{2 \times 9.81} \times H^{3/2} = 2.067\, H^{3/2}$$

or
$$H^{3/2} = \frac{0.1102}{2.067} = 0.0533$$

$\therefore$
$$H = (0.0533)^{2/3} = 0.1415 \text{ m} = \mathbf{141.5 \text{ mm}}. \quad \textbf{Ans.}$$

Problem 8.6. *A rectangular channel 2.0 m wide has a discharge of 250 litres per second, which is measured by a right- angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not to exceed 1.3 m. Take $C_d = 0.62$.*

Sol. Given :

Width of rectangular channel, $L = 2.0$ m

Discharge, $Q = 250$ lit/s $= 0.25$ m^3/s

Depth of water in channel $= 1.3$ m

Let the height of water over *V*-notch $= H$

The rate of flow through *V*-notch is given by equation (8.2) as

$$Q = \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\frac{\theta}{2} \times H^{5/2}$$

where $C_d = 0.62$, $\theta = 90°$

$\therefore$
$$Q = \frac{8}{15} \times .62 \times \sqrt{2 \times 9.81} \times \tan\frac{90}{2} \times H^{5/2}$$

or
$$0.25 = \frac{8}{15} \times .62 \times 4.429 \times 1 \times H^{5/2}$$

or
$$H^{5/2} = \frac{.25 \times 15}{8 \times .62 \times 4.429} = 0.1707$$

$\therefore$
$$H = (.1707)^{2/5} = (.1707)^{0.4} = 0.493 \text{ m}$$

Position of apex of the notch from the bed of channel

= depth of water in channel-height of water over *V*-notch

$= 1.3 - .493 =$ **0.807 m. Ans.**

Flow Through Pipes

11.1. INTRODUCTION

In the chapters 9 and 10, laminar flow and turbulent flow have been discussed. We have seen that when the Reynold number is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynold number is more than 4000, the flow is known as turbulent flow. In this chapter, the turbulent flow of fluids through pipes running full will be considered. If the pipes are partially full as in the case of sewer lines, the pressure inside the pipe is same and equal to atmospheric pressure. Then the flow of fluid in the pipe is not under pressure. This case will be taken in the chapter of flow of water through open channels. Here we will consider flow of fluids through pipes under pressure only.

11.2. LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

   This is due to friction and it is calculated by the following formulae :

   (*a*) Dracy-Weisbach Formula

   (*b*) Chezy's Formula

2. Minor Energy Losses

   This is due to

   (*a*) Sudden expansion of pipe

   (*b*) Sudden contraction of pipe

   (*c*) Bend in pipe

   (*d*) Pipe fittings etc.

   (*e*) An obstruction in pipe.

11.3. LOSS OF ENERGY (OR HEAD) DUE TO FRICTION

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g} \quad \text{...(11.1)}$$

where h_f = loss of head due to friction

f = Co-efficient of friction which is a function of Reynold number

$= \dfrac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \dfrac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

(*b*) **Chezy's Formula for loss of head due to friction in pipes.** Refer to chapter 10 article 10.3.1 in which expression for loss of head due to friction in pipes is derived. The equation (*iii*) of article 10.3.1, is

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad \text{...(11.2)}$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,

A = area of cross-section of pipe, L = Length of pipe,

and V = mean velocity of flow.

Now the ratio of $\frac{A}{P} \left(= \frac{\text{Area of flow}}{\text{Perimeter (wetted)}} \right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $$m = \frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4}$$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (11.2), we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \quad \text{or} \quad V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore \quad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{h_f}{L}} \qquad \text{...(11.3)}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation (11.3), we get

$$V = C\sqrt{mi} \qquad \text{...(11.4)}$$

Equation (11.4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

Problem 11.1. *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60. Take ν for water = 0.01 stoke.*

Sol. Given :

Dia. of pipe, d = 300 mm = 0.30 m

Length of pipe, L = 50 m

Velocity of flow, V = 3 m/s

Chezy's constant, C = 60

Kinematic viscosity, ν = 0.01 stoke = 0.01 cm^2/s

= 0.01×10^{-4} m^2/s.

(*i*) **Darcy Formula** is given by equation (11.1) as

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynold number, R_e

But R_e is given by $\quad R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$

∴ Value of $\quad f = \frac{0.079}{R_e^{1/4}} = \frac{.079}{(9 \times 10^5)^{1/4}} = .00256$

∴ Head lost, $\quad h_f = \frac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} = \mathbf{.7828\ m.}$ **Ans.**

(*ii*) **Chezy's Formula.** Using equation (11.4)

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \frac{d}{4} = \frac{0.30}{4} = 0.075$ m

∴ $\quad 3 = 60\sqrt{.075 \times i} \quad$ or $\quad i = \left(\frac{3}{60}\right)^2 \times \frac{1}{.075} = 0.0333$

But $\quad i = \frac{h_f}{L} = \frac{h_f}{50}$

Equating the two values of i, we have $\frac{h_f}{50} = .0333$

∴ $\quad h_f = 50 \times .0333 = \mathbf{1.665\ m.}$ **Ans.**

Problem 11.2. *Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of C = 50 in Chezy's formulae.*

Sol. Given :

Length of pipe, $\quad L = 2000$ m

Discharge, $\quad Q = 200$ litre/s $= 0.2\ m^3/s$

Head lost due to friction, $\quad h_f = 4$ m

Value of Chezy's constant, $\quad C = 50$

Let the diameter of pipe $\quad = d$

Velocity of flow, $\quad V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$

Hydraulic mean depth, $\quad m = \frac{d}{4}$

Loss of head per unit length, $\quad i = \frac{h_f}{L} = \frac{4}{2000} = .002$

Chezy's formula is given by equation (11.4) as $V = C\sqrt{mi}$

Substituting the values of V, m, i and C, we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times .002} \quad \text{or} \quad \sqrt{\frac{d}{4} \times .002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{.00509}{d^2}$$

Squaring both sides, $\frac{d}{4} \times .002 = \frac{.00509^2}{d^4} = \frac{.0000259}{d^4}$ or $d^5 = \frac{4 \times .0000259}{.002} = 0.0518$

$\therefore$ $d = \sqrt[5]{0.0518} = (.0518)^{1/5} = 0.553$ m = **553 mm. Ans.**

Problem 11.3. *A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 litres per sec. Find the head lost due to friction for a length of 50 m of the pipe.*

Sol. Given :

Kinematic viscosity, $\nu = 0.4$ stoke $= 0.4$ cm^2/s $= .4 \times 10^{-4}$ m^2/s

Dia. of pipe, $d = 300$ mm $= 0.30$ m

Discharge, $Q = 300$ litres/s $= 0.3$ m^3/s

Length of pipe, $L = 50$ m

Velocity of flow, $V = \frac{Q}{\text{Area}} = \frac{0.3}{\frac{\pi}{4}(0.3)^2} = 4.24$ m/s

$\therefore$ Reynold number, $R_e = \frac{V \times d}{\nu} = \frac{4.24 \times 0.30}{0.4 \times 10^{-4}} = 3.18 \times 10^4$

As R_e lies between 4000 and 100,000, the value of f is given by

$$f = \frac{.079}{(R_e)^{1/4}} = \frac{.079}{(3.18 \times 10^4)^{1/4}} = .00591$$

$\therefore$ Head lost due to friction, $h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g} = \frac{4 \times .00591 \times 50 \times 4.24^2}{0.3 \times 2 \times 9.81} =$ **3.61 m. Ans.**

Problem 11.4. *An oil of sp. gr. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take $\nu = .29$ stokes.*

Sol. Given :

Sp. gr. of oil, $S = 0.7$

Dia. of pipe, $d = 300$ mm $= 0.3$ m

Discharge, $Q = 500$ litres/s $= 0.5$ m^3/s

Length of pipe, $L = 1000$ m

Velocity, $V = \frac{Q}{\text{Area}} = \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073$ m/s

$\therefore$ Reynold number, $R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times (10)^4$

$\therefore$ Co-efficient of friction, $f = \frac{.079}{R_e^{1/4}} = \frac{0.79}{(7.316 \times 10^4)^{1/4}} = .0048$

$\therefore$ Head lost due to friction, $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18$ m

Power required $= \frac{\rho g . Q . h_f}{1000}$ kW

where ρ = density of oil $= 0.7 \times 1000 = 700$ kg/m^3

$\therefore$ Power required $= \frac{700 \times 9.81 \times 0.5 \times 163.18}{1000} =$ **560.28 kW. Ans.**

Problem 11.5. *Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 'f' = 0.009 in the formula* $h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$.

Sol. Given :

Dia. of pipe, $d = 200 \text{ mm} = 0.20 \text{ m}$

Length of pipe, $L = 500 \text{ m}$

Difference of pressure head, $h_f = 4 \text{ m of water}$

$f = .009$

Using equation (11.1), we have $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$

or

$$4.0 = \frac{4 \times .009 \times 500 \times V^2}{0.2 \times 2 \times 9.81} \quad \text{or} \quad V^2 = \frac{4.0 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

∴ $V = \sqrt{0.872} = 0.9338 \simeq 0.934 \text{ m/s}$

∴ Discharge, $Q = \text{velocity} \times \text{area}$

$$= 0.934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} (0.2)^2$$

$= 0.0293 \text{ m}^3\text{/s} =$ **29.3 litres/s. Ans.**

Problem 11.6. *Water is flowing through a pipe of diameter 200 mm with a velocity of 3 m/s. Find the head lost due to friction for a length of 5 m if the co-efficient of friction is given by* $f = .002 + \frac{.09}{R_e^{0.3}}$, *where* R_e *is Reynold number. The kinematic viscosity of water = .01 stoke.*

Sol. Given :

Dia. of pipe, $d = 200 \text{ mm} = 0.20 \text{ m}$

Velocity, $V = 3 \text{ m/s}$

Length, $L = 5 \text{ m}$

Kinematic Viscosity, $\nu = 0.01 \text{ stoke} = .01 \times 10^{-4} \text{ m}^2\text{/s}$

∴ Reynold number, $R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{.01 \times 10^{-4}} = 6 \times 10^5$

Value of

$$f = .02 + \frac{0.9}{Re^{0.3}} = .02 + \frac{.09}{(6 \times 10^5)^{.3}} = .02 + \frac{0.9}{54.13}$$

$$= .02 + .00166 = 0.02166$$

∴ Head lost due to friction, $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4.0 \times .02166 \times 5.0 \times 3^2}{0.20 \times 2.0 \times 9.81}$

= **0.993 m of water. Ans.**

Problem 11.7. *An oil of sp. gr. 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 litres/s. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.*

Sol. Given :

Sp. gr. of oil $= 0.9$

Viscosity, $\mu = 0.06 \text{ poise} = \frac{0.06}{10} \text{ Ns/m}^2$

Dia. of pipe, $d = 200 \text{ mm} = 0.2 \text{ m}$

Discharge, $Q = 60 \text{ litres/s} = 0.06 \text{ m}^3\text{/s}$

Length, $L = 500 \text{ m}$

Density, $\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$\therefore$ Reynold number, $R_e = \dfrac{\rho V d}{\mu} = 900 \times \dfrac{V \times 0.2}{\frac{0.06}{10}}$

where $V = \dfrac{Q}{\text{Area}} = \dfrac{0.06}{\frac{\pi}{4} d^2} = \dfrac{0.06}{\frac{\pi}{4} (.2)^2} = 1.909 \text{ m/s} \simeq 1.91 \text{ m/s}$

$$\therefore \quad R_e = 900 \times \frac{1.91 \times 0.2 \times 10}{0.06} = 57300$$

As R_e lies between 4000 and 10^5, the value of co-efficient of friction, f is given by

$$f = \frac{0.079}{R_e^{0.25}} = \frac{0.079}{(57300)^{0.25}} = .0051$$

$\therefore$ Head lost due to friction, $h_f = \dfrac{4 \times f \times L \times V^2}{d \times 2g} = \dfrac{4 \times .0051 \times 500 \times 1.91^2}{0.2 \times 2 \times 9.81}$

$= $ **9.48 m of water. Ans.**

$\therefore$ Power required $= \dfrac{\rho g . Q . h_f}{1000} = \dfrac{900 \times 9.81 \times 0.06 \times 9.48}{1000} =$ **5.02 kW. Ans.**

11.4. MINOR ENERGY (HEAD) LOSSES

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

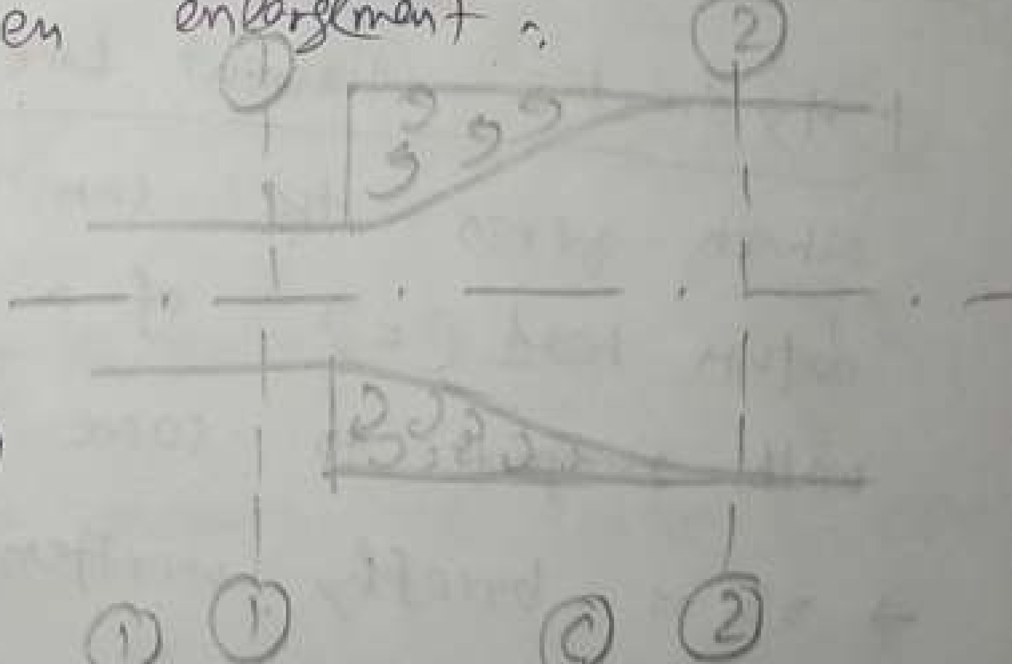
1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance to a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

Minor energy (head) losses

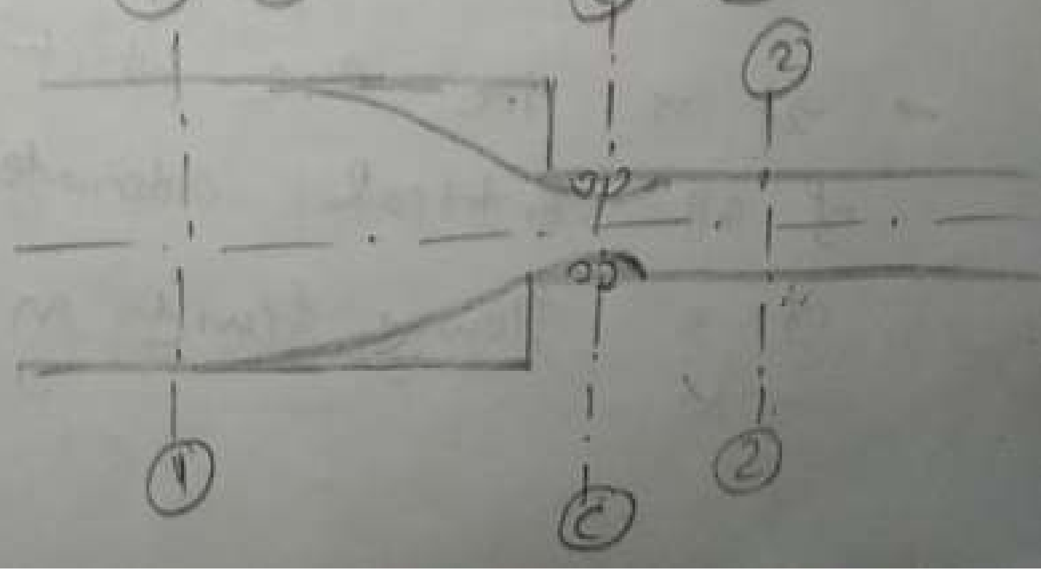
1. loss of head due to sudden enlargement :

$$h_e = \frac{(V_1 - V_2)^2}{2g}.$$



2. Loss of head due to sudden contraction

$$h_c = 0.5 \frac{V_2^2}{2g}$$



3. Loss of head at the entrance of a pipe:

→ This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir.

$h_i = 0.5 \frac{V^2}{2g}$ V = velocity of liquid in pipe

4. Loss of head at the exit of pipe:

→ This is the loss of head due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet or it is lost in the tank or reservoir.

$h_o = \frac{V^2}{2g}$ V = velocity at outlet of pipe.

5. Loss of head due to bend in pipe:

→ Loss of head in pipe due to bend

$h_b = \frac{KV^2}{2g}$, K = co-efficient of bend
V = velocity of flow

Hydraulic Gradient and Total Energy Line:

1. Hydraulic Gradient Line: (H.G.L.) It is defined as the line which gives the sum of pressure head $\left(\frac{P}{\rho g}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line

→ It is briefly written as H.G.L. (Hydraulic Gradient line).

→ It is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head of a flowing fluid in a pipe from the centre of pipe

9. Total Energy Line: (T.E.L.)

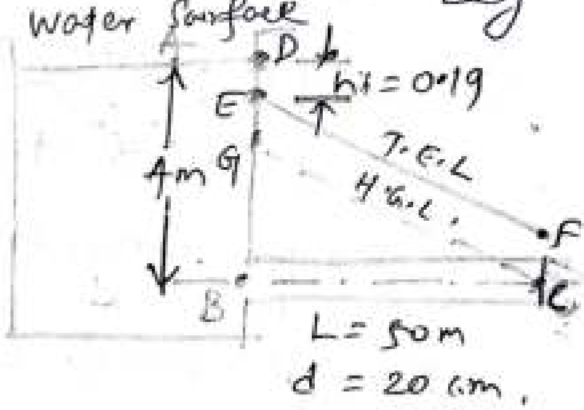
→ It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

→ It is also defined as the line which is obtained by joining tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.

Problem:

Determine the rate of flow of water through a pipe of dia. 20 cm. and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the centre of the pipe. Take $f = 0.009$, in the formula $h_f = \frac{4fLV^2}{2dg}$

Also draw H.G.L. & T.E.L.?



Ans, $d = 0.2\,m$, $L = 50\,m$,

$H = 4\,m$, $f = 0.009$.

→ Applying Bernoulli's eqn at the top of the water surface in the tank and at the outlet of pipe, we have

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{all losses}.$$

→ Considering datum line passing through the centre of pipe.

$$0 + 0 + 4 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$h_i = 0.5 \frac{V^2}{2g}$, $V_2 = V$:= velocity in pipe.

$$4 = \frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{4fLV^2}{2gd} =$$

$\Rightarrow 4 = \frac{v^2}{2g}\left(1 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2}\right) = \frac{v^2}{2g}(1 + 0.5 + 9)$

$= 10.5 \times \frac{v^2}{2g}$.

$\Rightarrow v = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734$ m/sec.

$Q = A \times V = \pi/4 \times 0.2^2 \times 2.734 = 0.08589$ m³/sec

$= 85.89$ litres/sec.

h_i = head lost at the entrance of pipe

$= 0.5 \times \frac{v^2}{2g} = \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19$ m

h_f = head loss due to friction

$= \frac{4fLv^2}{2dg} = \frac{4 \times 0.009 \times 50 \times 2.734^2}{2 \times 0.2 \times 9.81} = 3.428$ m.

→ Consider three points A, B, C on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown.

(a) Total energy line (T.E.L.), (Point D represents total energy at A)

1. Total energy at A $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 0 + 0 + 4 = 4$ m.

2. Total energy at B = total energy at A $- h_i = 4 - 0.19$
Point E, DE = h_i, represents total energy at inlet of pipe. $= 3.81$ m

3. Total energy at C $= \frac{P_c}{\rho g} + \frac{v_c^2}{2g} + z_c = 0 + \frac{v^2}{2g} + 0$

Point F, CF = 0.38 represents total energy at outlet of pipe

Join D to E, and E to F, DEF represents T.E.L. $= \frac{2.734^2}{2 \times 9.81} = 0.38$ m.

(b) Hydraulic Gradient line (H.G.L.):

→ It gives the sum of $\left(\frac{p}{\rho g} + z\right)$ with reference to the datum line.

→ Hence H.G.L. is obtained by subtracting $\frac{v^2}{2g}$ from T.E.L. At outlet total energy $= \frac{v^2}{2g}$.

→ From C draw a line CG parallel to EF.
CG represents H.G.L.

Problem 11.16. *Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take* $f = .009$ *in the formula* $h_f = \dfrac{4 . f . L . V^2}{d \times 2g}$.

Sol. Dia. of pipe, $d = 20 \text{ cm} = 0.20 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Height of water, $H = 4 \text{ m}$

Co-efficient of friction, $f = .009$

Let the velocity of water in pipe $= V$ m/s.

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have [Taking point 1 on the top and point 2 at the outlet of pipe].

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{all losses}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

or

$$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

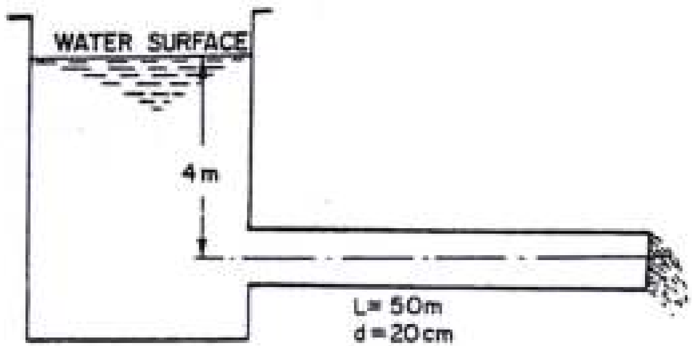


Fig. 11.4

But the velocity in pipe $= V$, $\therefore \quad V = V_2$

$$\therefore \quad 4.0 = \frac{V^2}{2g} + h_i + h_f \quad \text{...(i)}$$

From equation (11.8), $h_i = 0.5 \frac{V^2}{2g}$ and h_f from equation (11.1) is given as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

Substituting these values, we have

$$4.0 = \frac{V^2}{g} + \frac{0.5\, V^2}{2g} + \frac{4 \times f \,.\, L \,.\, V^2}{d \times 2g}$$

$$= \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times .009 \times 50}{0.2}\right] = \frac{V^2}{2g}[1.0 + 0.5 + 9.0] = 10.5 \times \frac{V^2}{2g}$$

$$\therefore \quad V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/sec}$$

$\therefore$ Rate of flow, $Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3/\text{s} =$ **85.89 litres/s. Ans.**

11.5. HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

11.5.1. Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

11.5.2. Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

Problem 11.22. *For the problem 11.16, draw the Hydraulic Gradient Line (H.G.L.) and Total Energy Line (T.E.L.).*

Sol. Given :

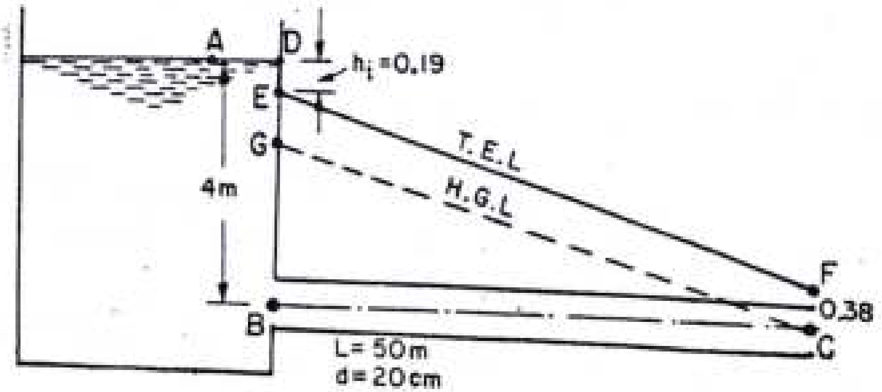
$$L = 50 \text{ m}, d = 200 \text{ mm} = 0.2 \text{ m}$$

$$H = 4 \text{ m}, \quad f = .009$$

Velocity, V through pipe is calculated in problem 11.16 and its value is $V = 2.734$ m/s

Now $\quad h_i$ = Head lost at entrance of pipe

$$= 0.5\frac{V^2}{2g} = \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19 \text{ m}$$



and

$$h_f = \text{Head loss due to friction}$$

$$= \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{0.2 \times 2 \times 9.81} = 3.428 \text{ m.}$$

(*a*) **Total Energy Line (T.E.L.).** Consider three points, A, B and C on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in Fig. 11.8. Let us find total energy at these points, taking the centre of pipe as reference line.

1. Total energy at $A = \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4$ m

2. Total energy at B = Total energy at $A - h_i = 4.0 - 0.19 = 3.81$ m

3. Total energy at $C = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c = 0 + \frac{V^2}{2g} + 0 = \frac{2.734^2}{2 \times 9.81} = 0.38$ m

Hence total energy line will coincide with free surface of water in the tank. At the inlet of the pipe, it will decrease by h_i (= 0.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in the Fig. 11.8,

(*i*) Point D represents total energy at A

(*ii*) Point E, where $DE = h_i$, represents total energy at inlet of the pipe

(*iii*) Point F, where $CF = 0.38$ represents total energy at outlet of pipe.

Join D to E and E to F. Then DEF represents the total energy line.

(*b*) **Hydraulic Gradient Line (H.G.L.).** H.G.L. gives the sum of $(p/w + z)$ with reference to the datum-line. Hence hydraulic gradient line is obtained by subtracting $\frac{V^2}{2g}$ from total energy line. At outlet of the pipe, total energy $= \frac{V^2}{2g}$. By subtracting $\frac{V^2}{2g}$ from total energy at this point, we shall get point C, which lies on the centre line of pipe. From C, draw a line CG parallel to EF. Then CG represents the hydraulic gradient line.

Impact of Jets and Jet Propulsion

17.1. INTRODUCTION

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion or from impulse-momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary or moving. In this chapter, the following cases of the impact of jet *i.e.*, the force exerted by the jet on a plate, will be considered :

(*a*) Force exerted by the jet on a stationary plate when

1. Plate is vertical to the jet, 2. Plate is inclined to the jet, and
3. Plate is curved.

(*b*) Force exerted by the jet on a moving plate, when

1. Plate is vertical to the jet, 2. Plate is inclined to the jet, and
3. Plate is curved.

17.2. FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in Fig. 17.1.

Let V = velocity of the jet, d = diameter of the jet,

a = area of cross-section of the jet $= \frac{\pi}{4} d^2$.

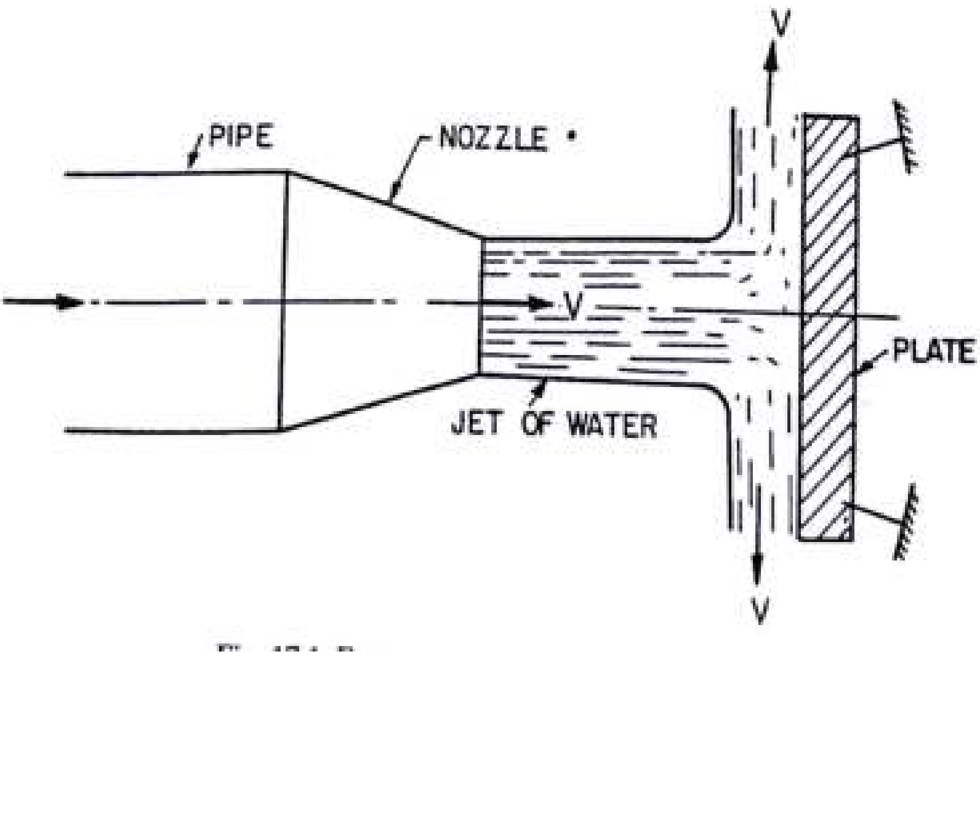


Fig. 17.1. F

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{final velocity of jet after striking})$$

$$= \rho a V [V - 0] \qquad (\because \ \text{mass/sec} = \rho \times aV)$$

$$= \rho a V^2 \qquad \text{...(17.1)}$$

For deriving above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated then final minus initial velocity is taken. But if the force exerted by the jet on the plate is to be calculated, then initial velocity minus final velocity is taken.

Note. In equation (17.1), if the value of density (ρ) is taken in S.I. units (*i.e.* kg/m^3), the force (F_x) will be in Newton (N). The value of ρ for water in S.I. units is equal to 1000 kg/m^3.

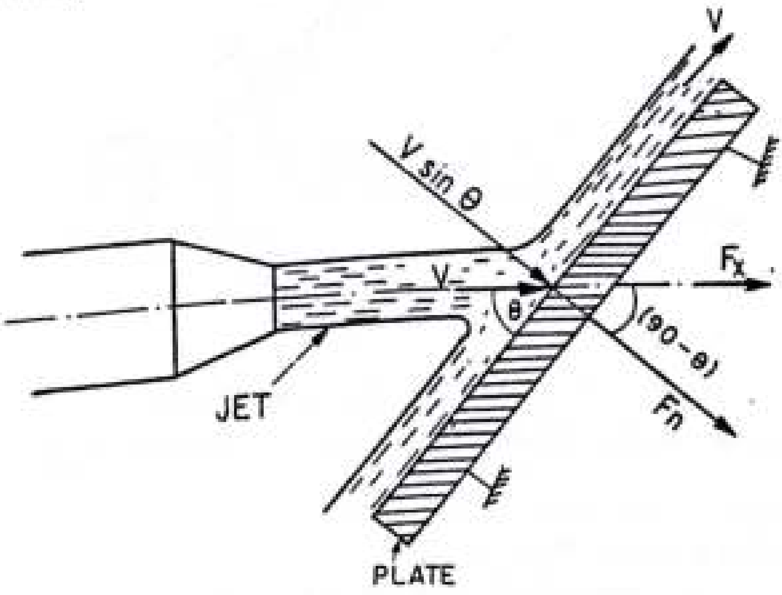
17.2.1. Force Exerted by a Jet on Stationary Inclined Flat Plate. Let a jet of water, coming out from the nozzle, strikes an inclined flat plate as shown in Fig. 17.2.

Let V = Velocity of jet in the direction of x,

θ = Angle between the jet and plate,

a = Area of cross-section of the jet.

Then mass of water per sec striking the plate = $\rho \times aV$.



If the plate is smooth and if it is assumed that there is no loss of energy due to impact of the jet, then jet will move over the plate after striking with a velocity equal to initial velocity *i.e.*, with a velocity V. Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n.

Then F_n = mass of jet striking per second × [Initial velocity of jet before striking in the direction of n – Find velocity of jet after striking in the direction of n]

$$= \rho a V[V \sin \theta - 0] = \rho a V^2 \sin \theta \quad \text{...(17.2)}$$

This force can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of flow. Then we have,

F_x = component of F_n in the direction of flow

$$= F_n \cos (90° - \theta) = F_n \sin \theta = \rho A V^2 \sin \theta \times \sin \theta \quad (\because \; F_n = \rho a V^2 \sin \theta)$$
$$= \rho A V^2 \sin^2 \theta \quad \text{...(17.3)}$$

And, F_y = component of F_n, perpendicular to flow

$$= F_n \sin (90° - \theta) = F_n \cos \theta = \rho A V^2 \sin \theta \cos \theta. \quad \text{...(17.4)}$$

17.2.2. Force exerted by a jet on stationary curved plate

(A) Jet Strikes the curved plate at the centre. Let a jet of water strikes a fixed curved plate at the centre as shown in Fig. 17.3. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet = $- V \cos \theta$.

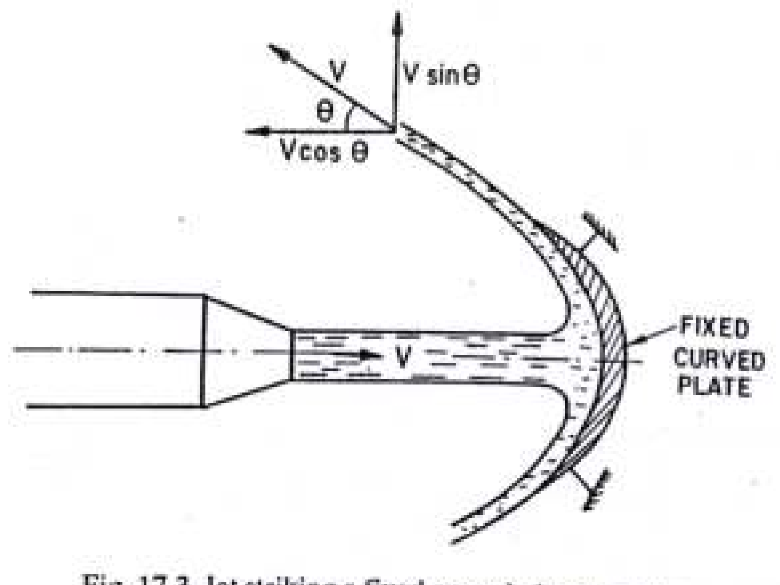


Fig. 17.3. Jet striking a fixed curved plate at centre.

(– ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

Component of velocity perpendicular to the jet = $V \sin \theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $- V \cos \theta$

$\therefore$ $$F_x = \rho a V[V - (-V\cos\theta)] = \rho a V[V + V\cos\theta]$$
$$= \rho a V^2 [1 + \cos\theta] \quad ...(17.5)$$

Similarly, $$F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}]$$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V\sin\theta$

$\therefore$ $$F_y = \rho a V[0 - V\sin\theta] = -\rho a V^2 \sin\theta \quad ...(17.6)$$

– ve sign means the force is acting in the downward direction. In this case the angle of deflection of the jet

$$= (180° - \theta) \quad ...[17.6\,(a)]$$

(B) Jet strikes the curved plate at one end tangentially when the plate is Symmetrical. Let the jet strikes the curved fixed plate at one end tangentially as shown in Fig. 17.4. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = Velocity of jet of water

θ = Angle made by jet with x-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the directions of x and y are

$$F_x = (\text{mass/sec}) \times [V_{1x} - V_{2x}]$$
$$= \rho a V[V\cos\theta - (-V\cos\theta)]$$
$$= \rho a V[V\cos\theta + V\cos\theta]$$
$$= 2\rho a V^2 \cos\theta \quad ...(17.7)$$
$$F_y = \rho a V[V_{1y} - V_{2y}]$$
$$= \rho a V[V\sin\theta - V\sin\theta] = 0.$$

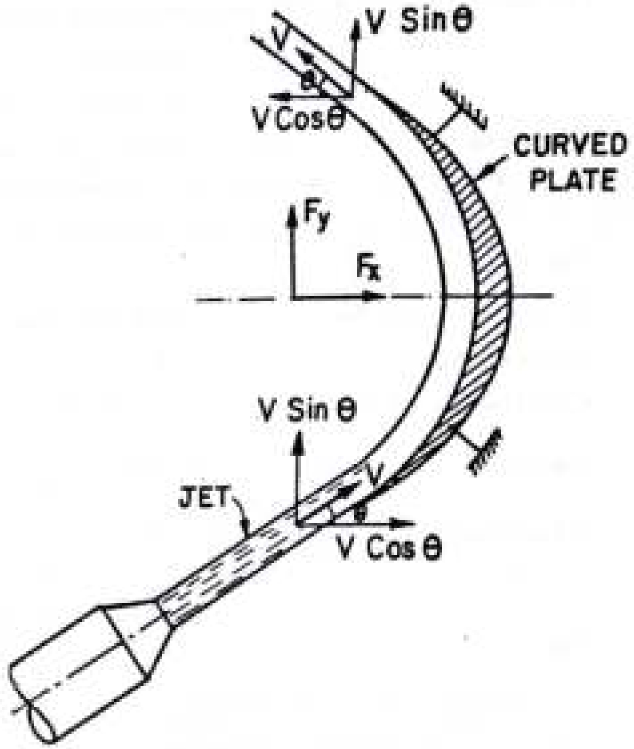


Fig. 17.4. Jet striking curved fixed plate at one end.

(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical. When the curved plate is unsymmetrical about x-axis, then the angles made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis,

ϕ = angle made by tangent at outlet tip with x-axis.

The two components of the velocity at inlet are

$$V_{1x} = V\cos\theta \text{ and } V_{1y} = V\sin\theta$$

The two components of the velocity at outlet are

$$V_{2x} = -V\cos\phi \text{ and } V_{2y} = V\sin\phi$$

$\therefore$ The forces exerted by the jet of water in the directions of x and y are

$$F_x = \rho a V[V_{1x} - V_{2x}] = \rho a V[V\cos\theta - (-V\cos\phi)]$$
$$= \rho a V[V\cos\theta + V\cos\phi] = \rho a V^2[\cos\theta + \cos\phi] \quad ...(17.8)$$

$$F_y = \rho a V [V_{1y} - V_{2y}] = \rho a V [V \sin\theta - V \sin\phi]$$
$$= \rho a V^2 [\sin\theta - \sin\phi]. \quad ...(17.9)$$

Problem 17.1. *Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 20 m/s.*

Sol. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 20$ m/s.

The force exerted by the jet of water on a stationary vertical plate is given by equation (17.1) as

$$F = \rho a V^2 \qquad \text{where } \rho = 1000 \text{ kg/m}^3$$

$\therefore$ $F = 1000 \times .004417 \times 20^2 \text{ N} = $ **1766.8 N. Ans.**

Problem 17.2. *Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre of nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.*

Sol. Given :

Diameter of nozzle, $d = 100 \text{ mm} = 0.1 \text{ m}$

Head of water, $H = 100$ m

Co-efficient of velocity, $C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Theoretical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

But $C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$

$\therefore$ Actual velocity of jet of water, $V = C_v \times V_{th} = 0.95 \times 44.294 = 42.08$ m/s.

Force on a fixed vertical plate is given by equation (17.1) as

$$F = \rho a V^2 = 1000 \times .007854 \times 42.08^2 \qquad (\because \text{ In S.I. units } \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

$= 13907.2 \text{ N} = $ **13.9 kN. Ans.**

Problem 17.3. *A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.*

Sol. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet, $V = 25$ m/s.

Angle between jet and plate, $\theta = 60°$

(*i*) The force exerted by the jet of water in the direction normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin\theta$$

$= 1000 \times .004417 \times 25^2 \times \sin 60° = $ **2390.7 N. Ans.**

(*ii*) The force in the direction of the jet is given by equation (17.3),

$$F_x = \rho a V^2 \sin^2 \theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin^2 60 = \mathbf{2070.4\ N.} \quad \textbf{Ans.}$$

Problem 17.4. *A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.*

Sol. Given :

Diameter of jet, $d = 50\ \text{mm} = 0.05\ \text{m}$

∴ Area, $a = \frac{\pi}{4}(.05)^2 = .001963\ \text{m}^2$

Angle, $\theta = 30°$

Force in the direction of jet, $F_x = 1471.5$ N

Force in the direction of jet is given by equation (17.3) as $F_x = \rho a V^2 \sin^2 \theta$

As the force is given in Newton, the value of ρ should be taken equal to 1000 kg/m³.

∴ $$1471.5 = 1000 \times .001963 \times V^2 \times \sin^2 30 = .05\ V^2$$

∴ $$V^2 = \frac{150}{.05} = 3000.0$$

$$V = 54.77\ \text{m/s}$$

∴ Discharge, Q = Area × Velocity

$$= .001963 \times 54.77 = 0.1075\ \text{m}^3/\text{s} = \mathbf{107.5\ litres/s.} \quad \textbf{Ans.}$$

Problem 17.5. *A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.*

Sol. Given :

Diameter of the jet, $d = 50\ \text{mm} = 0.05\ \text{m}$

∴ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963\ \text{m}^2$

Velocity of jet, $V = 40$ m/s

Angle of deflection $= 120°$

From equation [17.6 (*a*)], the angle of deflection = 180° − θ

∴ 180° − θ = 120 or θ = 180° − 120° = 60°

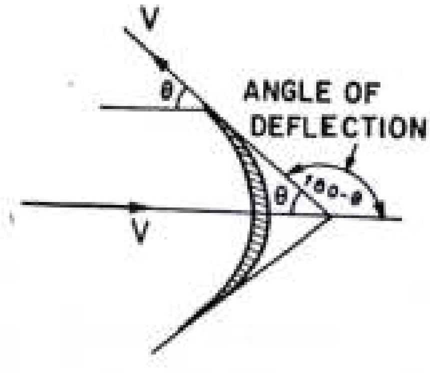


Fig. 17.5

Force exerted by the jet on the curved plate in the direction of the jet is given by equation (17.5) as

$$F_x = \rho a V^2 [1 + \cos \theta]$$

$$= 1000 \times .001963 \times 40^2 \times [1 + \cos 60°] = \mathbf{4711.15\ N.} \quad \textbf{Ans.}$$

Problem 17.6. *A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.*

Sol. Given :

Diameter of the jet, $d = 75\ \text{mm} = 0.075\ \text{m}$

∴ Area, $a = \frac{\pi}{4}(.075)^2 = .004417\ \text{m}^2$

Velocity of jet, $V = 30$ m/s

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of x is given by equation (17.8) and in the direction of y by equation (17.9),

$$\therefore \quad F_x = \rho a V^2 [\cos\theta + \cos\phi]$$

$$= 1000 \times .004417 [\cos 30 + \cos 20] \times 30^2 = \mathbf{7178.2 \text{ N.}} \quad \textbf{Ans.}$$

and

$$F_y = \rho a V^2 [\sin\theta - \sin\phi]$$

$$= 1000 \times .004417 [\sin 30 - \sin 20] \times 30^2 = \mathbf{628.13 \text{ N.}} \quad \textbf{Ans.}$$

Angle of swing, or angle made by deflected plate with the vertical, $\theta = 30°$

Dia. of the jet, $d = 25 \text{ mm} = 0.025 \text{ m}$

$\therefore$ Area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025^2) \text{ m}^2$

Velocity of jet, $V = 6 \text{ m/s}$

Let W = Weight of plate

Using equation (17.10), we get $\sin \theta = \frac{\rho \times a \times V^2}{W}$

$$\therefore \quad W = \frac{\rho^* \times a \times V^2}{\sin \theta} = \frac{1000 \times \left(\frac{\pi}{4} \times 0.025^2\right) \times 6^2}{\sin 30°} = \mathbf{35.33 \text{ N.}} \quad \textbf{Ans.}$$

17.4. FORCE EXERTED BY A JET ON MOVING PLATES

The following cases of the moving plates will be considered :

1. Flat vertical plate moving in the direction of the jet and away from the jet,
2. Inclined plate moving in the direction of the jet, and
3. Curved plate moving in the direction of the jet or in the horizontal direction.

17.4.1. Force on Flat vertical plate moving in the direction of Jet. Fig. 17.10 shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity of the jet (absolute),

a = Area of cross-section of the jet,

u = Velocity of the flat plate.

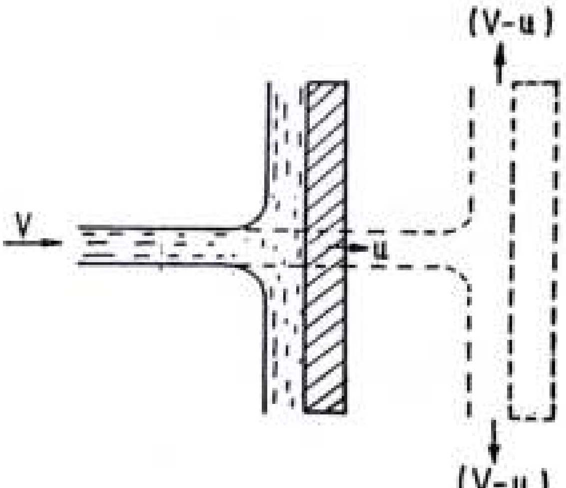


Fig. 17.10. Jet striking a flat vertical moving plate.

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence relative velocity of the jet with respect to plate

$= (V - u)$

Mass of water striking the plate per sec

$= \rho \times$ Area of jet $\times$ Velocity with which jet strikes the plate

$= \rho a \times [V - u]$

$\therefore$ Force exerted by the jet on the moving plate in the direction of the jet,

F_x = Mass of water striking per sec

$\times$ [Initial velocity with which water strikes − Final velocity]

$= \rho a(V - u)\,[(V - u) - 0]$ ($\because$ Final velocity in the direction of jet is zero)

$= \rho a(V - u)^2$ ...(17.11)

In this case, the work will be done by the jet on the plate, as plate is moving. For the stationary plates, the work done is zero.

*If $\rho = 1000 \text{ kg/m}^3$, then weight W will be in Netwon.

∴ Work done per second by the jet on the plate

$$= \text{Force} \times \frac{\text{Distance in the direction of force}}{\text{Time}}$$

$$= F_x \times u = \rho a(V-u)^2 \times u \qquad ...(17.12)$$

In equation (17.12), if the value of ρ for water is taken in S.I. units (*i.e.* 1000 kg/m^3), the work done will be in N m/s. The term $\frac{\text{'N m'}}{\text{s}}$ is equal to W (watt).

17.4.2. Force on the Inclined Plate moving in the direction of the Jet. Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.11.

Let V = Absolute velocity of jet of water

u = Velocity of the plate in the direction of jet

a = Cross-sectional area of jet

θ = Angle between jet and plate

Relative velocity of jet of water = $(V-u)$

∴ The velocity with which jet strikes = $(V-u)$

Mass of water striking per second

$$= \rho \times a \times (V-u)$$

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V-u)$.

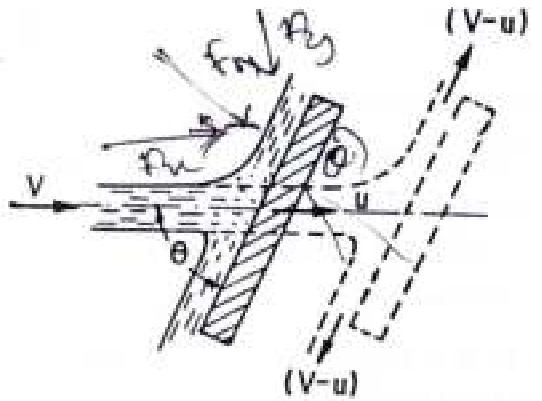


Fig. 17.11. Jet striking and inclined moving plate.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

F_n = Mass striking per second × [Initial velocity in the normal direction with which jet strikes – Final velocity]

$$= \rho a\,(V-u) \times [(V-u)\sin\theta - 0] = \rho a\,(V-u)^2 \sin\theta \qquad ...(17.13)$$

This normal force F_n is resolved into two component namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$$\therefore \quad F_x = F_n \sin\theta = \rho a\,(V-u)^2 \sin^2\theta \qquad ...(17.14)$$

$$F_y = F_n \cos\theta = \rho a\,(V-u)^2 \sin\theta\cos\theta \qquad ...(17.15)$$

∴ Work done per second by the jet on the plate

$$= F_x \times \text{Distance per second in the direction of } x$$

$$= F_x \times u = \rho a(V-u)^2 \sin^2\theta \times u = \rho a\,(V-u)^2 u \sin^2\theta \text{ N m/s.} \qquad ...(17.16)$$

Problem 17.11. *A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find :*

(i) the force exerted by the jet on the plate,

(ii) work done by the jet on the plate per second.

Sol. Given :

Diameter of the jet, $d = 10 \text{ cm} = 0.1 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Velocity of jet, $V = 15$ m/s

Velocity of the plate, $u = 6$ m/s.

(*i*) The force exerted by the jet on a moving flat vertical plate is given by equation (17.11),

$$F_x = \rho a (V - u)^2$$

$$= 1000 \times .007854 \times (15 - 6)^2 \text{ N} = \mathbf{636.17 \text{ N}}. \quad \textbf{Ans.}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = \mathbf{3817.02 \text{ Nm/s}}. \quad \textbf{Ans.}$$

Problem 17.12. *For the problem 17.11, find the power and efficiency of the jet.*

Sol. The given data from problem 17.11 is

$$a = .007854 \text{ m}^2, \quad V = 15 \text{ m/s}, u = 6 \text{ m/s}$$

Also work done per second by the jet = 3817.02 Nm/s

(*i*) Power of the jet in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{3817.02}{1000} = \mathbf{3.817 \text{ kW}}$. **Ans.**

(*ii*) Efficiency of the jet (η) $= \dfrac{\text{Output of the jet per second}}{\text{Input of the jet per second}}$ ...(*i*)

where output of jet/sec = Work done by jet per second = 3817.02 Nm/s

And input per second = Kinetic energy of the jet/sec

$$= \frac{1}{2}\left(\frac{\text{mass}}{\text{sec}}\right) V^2 = \frac{1}{2}(\rho a V) \times V^2 = \frac{1}{2}\rho a V^3$$

$$= \frac{1}{2} \times 1000 \times .007854 \times 15^3 \text{ Nm/s} = 13253.6 \text{ Nm/s}$$

$\therefore$ η of the jet $= \dfrac{3817.02}{13253.6} = 0.288 = \mathbf{28.8\%}$. **Ans.**

Problem 17.13. *A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate : (i) when the plate is stationary, and (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and the efficiency of the jet when the plate is moving.* (AMIE, Winter 1981)

Sol. Given :

Diameter of the jet, $d = 7.5 \text{ cm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \dfrac{\pi}{4}(.075)^2 = .004417 \text{ m}^2$

Angle between the jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet, $V = 30 \text{ m/s}$.

(*i*) When the plate is stationary, the normal force on the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin\theta = 1000 \times .004417 \times 30^2 \times \sin 45 = \mathbf{2810.96 \text{ N}}. \quad \textbf{Ans.}$$

(*ii*) When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given by equation (17.13) as

$$F_n = \rho a (V - u)^2 \sin\theta \qquad \text{where } u = 15 \text{ m/s}.$$

$$= 1000 \times .004417 \times (30 - 15)^2 \times \sin 45° = \mathbf{702.74 \text{ N}}. \quad \textbf{Ans.}$$

(*iii*) The power and efficiency of the jet when plate is moving is obtained as

Work done per second by the jet

= Force in the direction of jet

× Distance moved by the plate in the direction of jet/sec

$= F_x \times u$ where $F_x = F_n \sin\theta = 702.74 \times \sin 45 = 496.9$ N

Work done/sec $= 496.9 \times 15 = 7453.5$ Nm/s

$\therefore$ Power in kW $= \dfrac{\text{Work done per second}}{1000} = \dfrac{7453.5}{1000} = $ **7.453 kW. Ans.**

Efficiency of the jet $= \dfrac{\text{Output}}{\text{Input}} = \dfrac{\text{Work done per second}}{\text{Kinetic energy of the jet}}$

$$= \frac{7453.5}{\frac{1}{2}(\rho aV) \times V^2} = \frac{7453.5}{\frac{1}{2}\rho aV^3} = \frac{7453.5}{\frac{1}{2} \times 1000 \times .004417 \times 30^3}$$

$= 0.1249 \simeq 0.125 =$ **12.5%. Ans.**

17.4.3. Force on the Curved Plate when the Plate is Moving in the Direction of Jet. Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.12.

Let V = Absolute velocity of jet,

a = Area of jet,

u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate $= (V - u)$.

If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane $= (V - u)$.

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

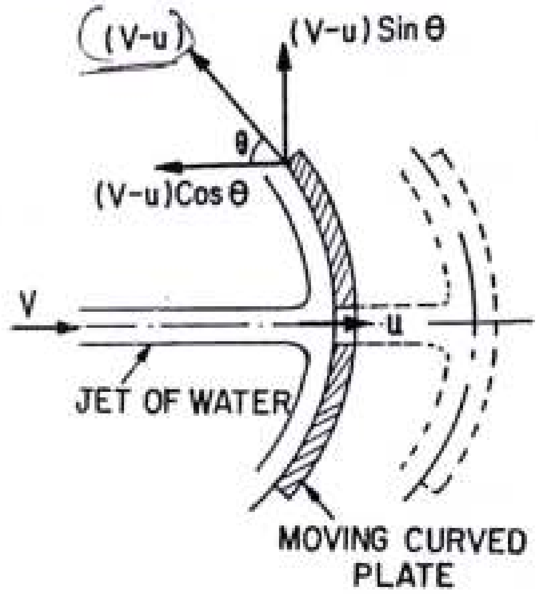


Fig. 17.12. Jet striking a curved moving plate.

Component of the velocity in the direction of jet

$= -(V - u) \cos \theta$

(– ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet $= (V - u) \sin \theta$.

Mass of the water striking the plate $= \rho \times a \times$ Velocity with which jet strikes the plate

$= \rho a(V - u)$

$\therefore$ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$= \rho a(V - u)[(V - u) - (-(V - u) \cos \theta)]$

$= \rho a\,(V - u)[(V - u) + (V - u) \cos \theta]$

$= \rho a(V - u)^2\,[1 + \cos \theta]$ ...(17.17)

Work done by the jet on the plate per second

$= F_x \times$ Distance travelled per second in the direction of x

$= F_x \times u = \rho a(V - u)^2\,[1 + \cos \theta] \times u$

$= \rho a(V - u)^2 \times u\,[1 + \cos \theta]$ ...(17.18)

Problem 17.14. *A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :*

(i) Force exerted on the plate in the direction of jet, *(ii) Power of the jet, and*

(iii) Efficiency of the jet.

Sol. Given :

Diameter of the jet, $d = 7.5 \text{ cm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = 0.004417$

Velocity of the jet, $V = 20 \text{ m/s}$

Velocity of the plate, $u = 8 \text{ m/s}$

Angle of deflection of the jet $= 165°$

$\therefore$ Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°.$$

(*i*) Force exerted by the jet on the plate in the direction of the jet is given by equation (17.17) as

$$F_x = \rho a(V - u)^2 (1 + \cos\theta)$$

$$= 1000 \times .004417 \times (20 - 8)^2 [1 + \cos 15] = \mathbf{1250.38\ N.} \quad \textbf{Ans.}$$

(*ii*) Work done by the jet on the plate per second

$$= F_x \times u = 1250.38 \times 8 = 10003.04 \text{ N m/s}$$

$\therefore$ Power of the jet $= \frac{10003.04}{1000} = \mathbf{10\ kW.} \quad \textbf{Ans.}$

(*iii*) Efficiency of the jet $= \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$

$$= \frac{1250.38 \times 8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times V^3}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times 20^3} = 0.564 = \mathbf{56.4\%.} \quad \textbf{Ans.}$$

17.4.4. Force exerted by a Jet of Water on an Un-Symmetrical moving Curved Plate when Jet strikes tangentially at one of the tips. Fig. 17.15 shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

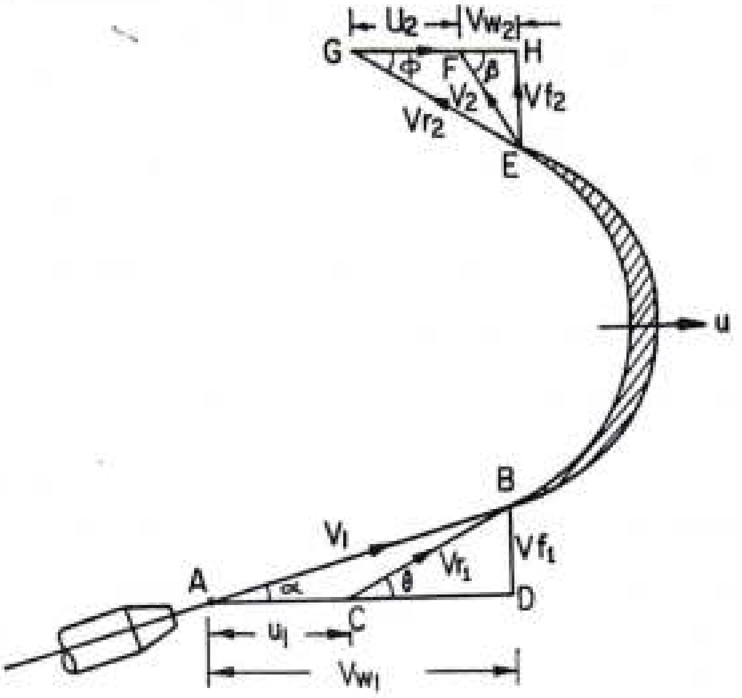


Fig. 17.15. Jet striking a moving curved vane at one of the tips.

Let V_1 = Velocity of the jet at inlet.

u_1 = Velocity of the plate (vane) as inlet.

V_{r_1} = Relative velocity of jet and plate at inlet.

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle.

θ = Angle made by the relative velocity (V_{r_1}) with the direction of motion at inlet also called vane angle at inlet.

V_{w_1} and V_{f_1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively.

V_{w_1} = It is also known as velocity of whirl at inlet.

V_{f_1} = It is also known as velocity of flow at inlet.

V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.

u_2 = Velocity of the vane at outlet.

V_{r_2} = Relative velocity of the jet with respect to the vane at outlet.

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet.

ϕ = Angle made by the relative velocity V_{r_2}, with the direction of motion of the vane at outlet and also called vane angle at outlet.

V_{w_2} and V_{f_2} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet.

V_{w_2} = It is also called the velocity of whirl at outlet.

V_{f_2} = Velocity of flow at outlet.

The triangles ABD and EGH are called the velocity triangles at inlet and outlet. These velocity triangles are drawn as given below :

1. Velocity Triangle at Inlet. Take any point A and draw a line $AB = V_1$ in magnitude and direction which means line AB makes an angle α with the horizontal line AD. Next draw a line $AC = u_1$ in magnitude. Join C to B. Then CB represents the relative velocity of the jet at inlet. If the loss of energy at inlet due to impact is zero, then CB must be in the tangential direction to the vane at inlet. From B draw a vertical line BD in the downward direction to meet the horizontal line AC produced at D.

Then BD = Represents the velocity of flow at inlet = V_{f_1}

AD = Represents the velocity of whirl at inlet = V_{w_1}

$\angle BCD$ = Vane angle at inlet = θ.

2. Velocity Triangle at Outlet. If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero. The water will be gliding over the surface of the vane with a relative velocity equal to V_{r_1} and will come out of the vane with a relative velocity V_{r_1}. This means that the relative velocity at outlet $V_{r_2} = V_{r_1}$. And also the relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw EG in the tangential direction of the vane at outlet and cut $EG = V_{r_2}$. From G, draw a line GF in the direction of vane at outlet and equal to u_2, the velocity of the vane at outlet. Join EF. Then EF represents the absolute velocity of the jet at outlet in magnitude and direction. From E draw a vertical line EH to meet the line GF produced at H. Then

EH = Velocity of flow at outlet = V_{f_2}

FH = Velocity of whirl at outlet = V_{w_2}

ϕ = Angle of vane at outlet

β = Angle made by V_2 with the direction of motion of vane at outlet.

If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal then we have

$u_1 = u_2 = u$ = Velocity of vane in the direction of motion and

$V_{r_1} = V_{r_2}$.

Now mass of water striking vane per sec = $\rho a V_{r_1}$ ...(*i*)

where a = Area of jet of water, V_{r_1} = Relative velocity at inlet.

$\therefore$ Force exerted by the jet in the direction of motion,

F_x = Mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion – Final velocity of jet in the direction of motion] ...(*ii*)

But initial velocity with which jet strikes the vane = V_{r_1}.

The component of this velocity in the direction of motion

$= V_{r_1} \cos\theta = (V_{w_1} - u_1)$ (See Fig. 17.15)

Similarly, the component of the relative velocity at outlet in the direction of motion = $-V_{r_2}\cos\phi$

$= -[u_2 + V_{w_2}]$

– ve sign is taken as the component of V_{r_2} in the direction of motion is in the opposite direction.

Substituting the equation (*i*) and all above values of the velocities in equation (*ii*), we get

$$F_x = \rho a V_{r_1}[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}] = \rho a V_{r_1}[V_{w_1} - u_1 + u_2 + V_{w_2}]$$

$$= \rho a V_{r_1}[V_{w_1} + V_{w_2}] \quad (\because u_1 = u_2) \quad ...(iii)$$

The equation (*iii*) is true only when angle β shown in Fig. 17.15 is an acute angle. If $\beta = 90°$, the $V_{w_2} = 0$, then equation (*iii*) becomes as,

$$F_x = \rho a V_{r_1}[V_{w_1}]$$

If β is an obtuse angle, the expression for F_x will become

$$F_x = \rho a V_{r_1}[V_{w_1} - V_{w_2}]$$

Thus in general, F_x is written as $F_x = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}]$ ...(17.19)

Work done per second on the vane by the jet

= Force × Distance per second in the direction of force

$$= F_x \times u = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u \quad ...(17.20)$$

∴ Work done per second per unit weight of fluid striking per second

$$= \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{\text{Weight of fluid striking/s}} \frac{\text{Nm/s}}{\text{N/s}} = \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{g \times \rho a V_{r_1}} \text{ Nm/N}$$

$$= \frac{1}{g}[V_{w_1} \pm V_{w_2}] \times u \text{ Nm/N} \quad ...(17.21)$$

Work done/sec per unit mass of fluid striking per second

$$= \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{\text{Mass of fluid striking/s}} \frac{\text{Nm/s}}{\text{kg/s}} = \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{\rho a V_{r_1}} \text{ Nm/kg}$$

$$= (V_{w_1} \pm V_{w_2}) \times u \text{ Nm/kg} \quad ...[17.21 (a)]$$

Note. The equation (17.21) gives the work done per unit weight whereas the equation [17.21 (*a*)] gives the work done per unit mass.

Problem 17.18. *A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane an outlet. Calculate :*

(*i*) *Vane angles, so that the water enters and leaves the vane without shock.*

(*ii*) *Work done per second per unit weight of water striking (or work done per unit weight of water striking) the vane per second.*

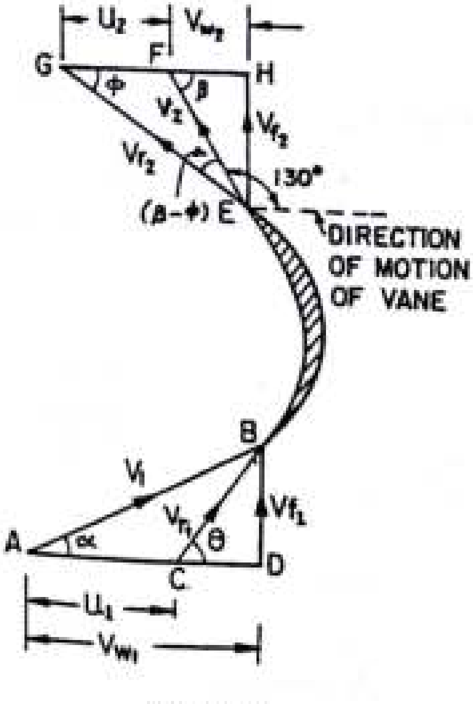


Fig. 17.16

Sol. Given :

Velocity of jet, $V_1 = 20$ m/s

Velocity of vane, $u_1 = 10$ m/s

Angle made by jet at inlet, with direction of motion of vane,

$\alpha = 20°$

Angle made by the leaving jet, with the direction of motion

$= 130°$

∴ $\beta = 180° - 130° = 50°$

In this problem, $u_1 = u_2 = 10$ m/s

$V_{r_1} = V_{r_2}$.

(*i*) **Vane Angles** means angle made by the relative velocities at inlet and outlet, *i.e.* θ and φ.

From Fig. 17.16, in ΔABD, we have $\tan\theta = \frac{BD}{CD}$

$$= \frac{V_{f_1}}{AD - AC} = \frac{V_{f_1}}{V_{w_1} - u_1} \quad \text{...(i)}$$

where $V_{f_1} = V_1 \sin\alpha = 20 \times \sin 20 = 6.84$ m/s

$V_{w_1} = V_1 \cos\alpha = 20 \times \cos 20 = 18.794$ m/s

$u_1 = 10$ m/s

$$\therefore \quad \tan\theta = \frac{6.84}{18.794 - 10} = .7778 \quad \text{or} \quad \theta = 37.875°$$

$$\therefore \quad \theta = 37° \ 52.5'. \quad \textbf{Ans.}$$

From, ΔABC, $$\sin\theta = \frac{V_{f_1}}{V_{r_1}} \quad \text{or} \quad V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37.875} = 11.14$$

$$\therefore \quad V_{r_2} = V_{r_1} = 11.14 \text{ m/s.}$$

From ΔEFG, applying sine rule, we have

$$\frac{V_{r_2}}{\sin(180° - \beta)} = \frac{u_2}{\sin(\beta - \phi)}$$

or $$\frac{11.14}{\sin\beta} = \frac{10}{\sin[\beta - \phi]} \text{ or } \frac{11.14}{\sin 50} = \frac{10}{\sin[50° - \phi]} \quad (\because \ \beta = 50°)$$

$$\therefore \quad \sin(50° - \phi) = \frac{10 \times \sin 50}{11.14} = 0.6876 = \sin 43.44°$$

$$\therefore \quad 50° - \phi = 43.44° \text{ or } \phi = 50° - 43.44° = 6.56°$$

$$\therefore \quad \phi = 6° \ 33.6.' \quad \textbf{Ans.}$$

(*ii*) Work done per second per unit weight of the water striking the vane per second is given by equation (17.21) as

$$= \frac{1}{g}[V_{w_1} + V_{w_2}] \times u \text{ Nm/N} \qquad (+ \text{ ve sign is taken as } \beta \text{ is an acute angle})$$

where $V_{w_1} = 18.794$ m/s, $V_{w_2} = GH - GF = V_{r_2}\cos\phi - u_2 = 11.14 \times \cos 6.56 - 10 = 1.067$ m/s

$u = u_1 = u_2 = 10$ m/s

$\therefore$ Work done per unit weight of water

$$= \frac{1}{9.81}[18.794 + 1.067] \times 10 \text{ Nm/N} = \textbf{20.24 Nm/N.} \quad \textbf{Ans.}$$

17.4.5. Force Exerted by a Jet of Water on a Series of Vanes. The force exerted by a jet of water on a *single* moving plate (which may be flat or curved) is not practically feasible. This case is only a theoretical one. In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown in Fig. 17.22. The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate. Thus each plate appears successively before the jet and the jet exerts force on each plate. The wheel starts moving at a constant speed.

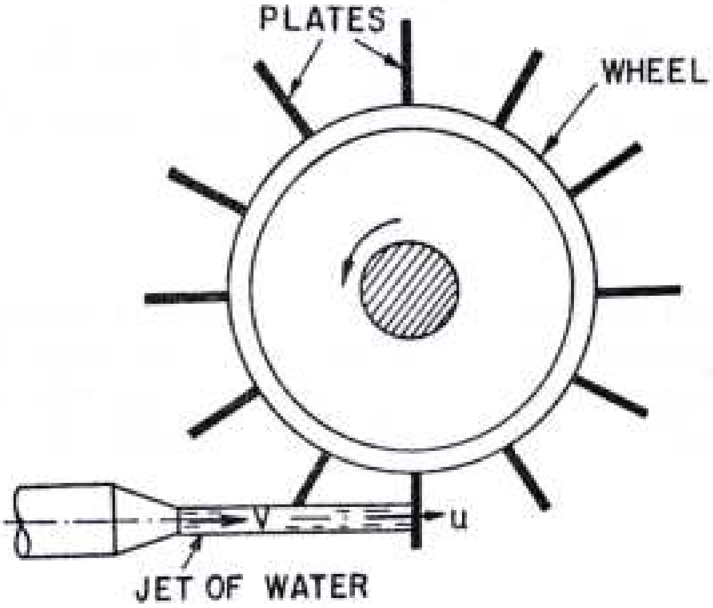


Fig. 17.22. Jet striking a series of vanes.

Let V = Velocity of jet,

d = Diameter of jet,

a = Cross-sectional area of jet,

$= \frac{\pi}{4} d^2$

u = Velocity of vane.

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered. Hence mass of water per second striking the series of plates = ρaV.

Also the jet strikes the plate with a velocity = $(V - u)$.

After striking, the jet moves tangential to the plate and hence the velocity component in the direction of motion of plate is equal to zero.

$\therefore$ The force exerted by the jet in the direction of motion of plate,

$$F_x = \text{Mass per second [Initial velocity} - \text{Final velocity]}$$

$$= \rho aV[(V - u) - 0] = \rho aV[V - u] \quad \text{...(17.22)}$$

Work done by the jet on the series of plates per second

$$= \text{Force} \times \text{Distance per second in the direction of force}$$

$$= F_x \times u = \rho a V [V - u] \times u$$

Kinetic energy of the jet per second

$$= \frac{1}{2} m V^2 = \frac{1}{2} (\rho a V) \times V^2 = \frac{1}{2} \rho a V^3$$

$\therefore$ Efficiency,

$$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho a V [V - u] \times u}{\frac{1}{2} \rho a V^3} = \frac{2u [V - u]}{V^2}. \quad \text{...(17.23)}$$

Condition for Maximum Efficiency. Equation (17.23) gives the value of the efficiency of the wheel. For a given jet velocity V, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V - u)}{V^2}\right] = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2uV - 2u^2}{V^2}\right] = 0$$

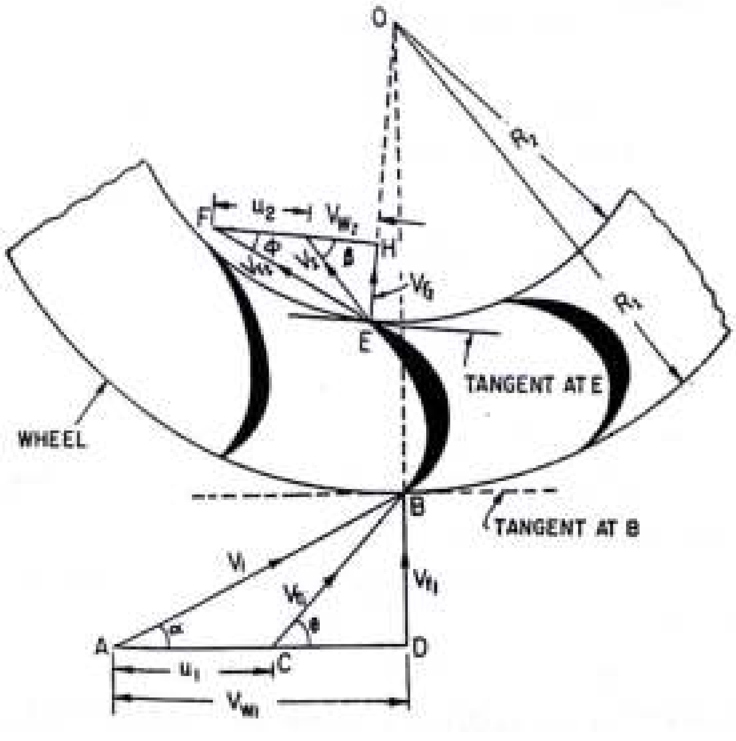
or

$$\frac{2V - 2 \times 2u}{V^2} = 0 \quad \text{or} \quad 2V - 4u = 0 \quad \text{or} \quad V = \frac{4u}{2} = 2u \quad \text{or} \quad u = \frac{V}{2}. \quad \text{...(17.24)}$$

Maximum Efficiency. Substituting the value of $V = 2u$ in equation (17.23), we get the maximum efficiency as

$$\eta_{max} = \frac{2u [2u - u]}{(2u)^2} = \frac{2u \times u}{2u \times 2u} = \frac{1}{2} \text{ or } 50\%. \quad \text{...(17.25)}$$

17.4.6. Force Exerted on a Series of Radial Curved Vanes. For a radial curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal. Consider a series of radial curved vanes mounted on a wheel as shown in Fig. 17.23. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.



Let R_1 = Radius of wheel at inlet of the vane,

R_2 = Radius of the wheel at the outlet of the vane,

ω = Angular speed of the wheel.

Then $u_1 = \omega R_1$ and $u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in Fig. 17.23.

The mass of water striking per second for a series of vanes

= Mass of water coming out from nozzle per second

= $\rho a V_1$, where a = Area of jet and V_1 = Velocity of jet.

Momentum of water striking the vanes in the tangential direction per sec at inlet

= Mass of water per second × Component of V_1 in the tangential direction

= $\rho a V_1 \times V_{w_1}$ ($\because$ Component of V_1 in tangential direction = $V_1 \cos\alpha = V_{w_1}$)

Similarly momentum of water at outlet per sec

= $\rho a V_1$ × component of V_2 in the tangential direction

= $\rho a V_1 \times (-V_2 \cos\beta) = -\rho a V_1 \times V_{w_2}$ ($\because$ $V_2 \cos\beta = V_{w_2}$)

– ve sign is taken as the velocity V_2 at outlet is in opposite direction.

Now angular momentum per second at inlet

= Momentum at inlet × Radius at inlet

= $\rho a V_1 \times V_{w_1} \times R_1$

Angular momentum per second at outlet

= Momentum of outlet × Radius at outlet

= $-\rho a V_1 \times V_{w_2} \times R_2$

Torque exerted by the water on the wheel,

T = Rate of change of angular momentum

= [Initial angular momentum per second – Final angular momentum per second]

$$= \rho a V_1 \times V_{w_1} \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2) = \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2]$$

Work done per second on the wheel

= Torque × Angular velocity = $T \times \omega$

$$= \rho a V_1 [V_{w_1} \times R_1 + V_{w_2} R_2] \times \omega = \rho a V_1 [V_{w_1} \times R_1 \times \omega + V_{w_2} R_2 \times \omega]$$

$$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} \times u_2] \quad (\because u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$$

If the angle β in Fig. 17.23 is an obtuse angle then work done per second will be given as

$$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2],$$

$\therefore$ The general expression for the work done per second on the wheel

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2] \quad \text{...(17.26)}$$

If the discharge is radial at outlet, then $\beta = 90°$ and work done becomes as

$$= \rho a V_1 [V_{w_1} u_1] \quad (\because V_{w_2} = 0) \quad \text{...(17.27)}$$

Efficiency of the Radial Curved Vane

Efficiency, $$\eta = \frac{\text{Work done per second}}{\text{Kinetic energy per second}} = \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2}(\text{mass/sec}) \times V_1^2}$$

$$= \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2 [V_{w_1} u_1 \pm V_{w_2} u_2]}{V_1^2}. \qquad ...(17.28)$$

Problem 17.24. *If in problem 17.23, the jet of water instead of striking a single plate, strikes a series of curved vanes, find for the data given in problem 17.23,*

(i) Force exerted by the jet on the vane in the direction of motion

(ii) Power exerted on the vane, and

(iii) Efficiency of the vane.

Sol. Given :

From problem 17.23, $V_1 = 15$ m/s, $u = u_1 = u_2 = 5$ m/s,

$\alpha = 0$, $a = .007854\ \text{m}^2$

$\phi = 45°$, $V_{w_1} = 15$ m/s and $V_{w_2} = 2.07$ m/s.

For the series of vanes, mass of water striking per second

= Mass of water coming out from nozzle

$= \rho a V_1 = 1000 \times .007854 \times 15 = 117.72$

(i) Force exerted by the jet on the vane in the direction of motion

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}] = 117.72 [15 + 2.07] = \mathbf{2009.5\ N}. \quad \textbf{Ans.}$$

(ii) Power of the vane in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{F_x \times u}{1000}\ \text{kW} = \frac{2009.5 \times 5}{1000} = \mathbf{10.05\ kW}. \quad \textbf{Ans.}$$

(iii) Efficiency, $$\eta = \frac{\text{Work done per second}}{\frac{1}{2}(\text{mass of water per sec}) \times V_1^2}$$

$$= \frac{2009.5 \times 5.0}{\frac{1}{2} \times 117.72 \times 15^2} = 0.7586 \text{ or } \mathbf{75.86\%}. \quad \textbf{Ans.}$$

Problem 17.25. *A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities at inlet and outlet and find :*

(a) the angles of vanes tips so that water enters and leaves without shock,

(b) the work done per unit weight of water entering the vanes, and

(c) the efficiency. (Fluid Power Engg., AMIE, Summer, 1984)

Sol. Given :

Velocity of jet, $V_1 = 35$ m/s

Velocity of vane, $u_1 = u_2 = 20$ m/s

Angle of jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet with the direction of motion of vanes = 120°

$\therefore$ Angle $\beta = 180° - 120° = 60°$

(*a*) *Angles of vanes tips.*

From inlet velocity triangle

$$V_{w_1} = V_1 \cos \alpha = 35 \cos 30° = 30.31 \text{ m/s}$$

$$V_{f_1} = V_1 \sin \alpha = 35 \sin 30° = 17.50 \text{ m/s}$$

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

$\therefore$ $$\theta = \tan^{-1} 1.697 = \mathbf{60°}. \quad \textbf{Ans.}$$

By sine rule, $$\frac{V_{r_1}}{\sin 90°} = \frac{V_{f_1}}{\sin \theta} \quad \text{or} \quad \frac{V_{r_1}}{1} = \frac{17.50}{\sin 60°}$$

$\therefore$ $$V_{r_1} = \frac{17.50}{.866} = 20.25 \text{ m/s}.$$

Now $$V_{r_2} = V_{r_1} = 20.25 \text{ m/s}$$

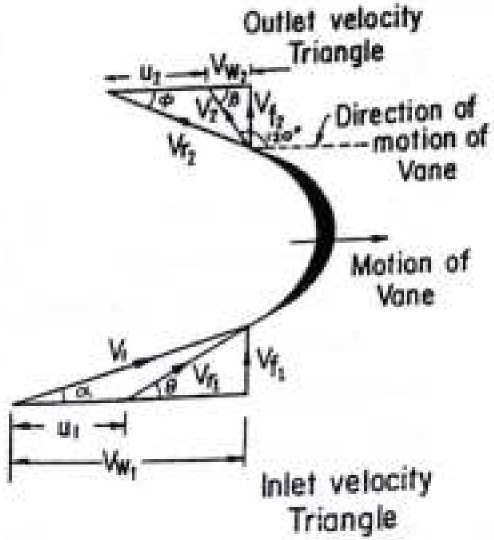


Fig. 17.23 (*a*)

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 120°} = \frac{u_2}{\sin (60° - \phi)} \quad \text{or} \quad \frac{20.25}{0.886} = \frac{20}{\sin (60° - \phi)}$$

$\therefore$ $$\sin (60° - \phi) = \frac{20 \times 0.866}{20.25} = 0.855 = \sin (58.75°)$$

$$60° - \phi = 58.75°$$

$\therefore$ $$\phi = 60° - 58.75 = \mathbf{1.25°}. \quad \textbf{Ans.}$$

(*b*) Work done per unit weight of water entering $= \dfrac{1}{g} (V_{w_1} + V_{w_2}) \times u_1$ ...(*i*)

$$V_{w_1} = 30.31 \text{ m/s and } u_1 = 30 \text{ m/s}$$

The value of V_{w_2} is obtained from outlet velocity triangle

$$V_{w_2} = V_{r_2} \cos \phi - u_2 = 20.25 \cos 1.25° - 20.0 = 0.24 \text{ m/s}$$

$\therefore$ Work done/unit weight $= \dfrac{1}{9.81} [30.31 + 0.24] \times 20 = \mathbf{62.28 \text{ Nm/N}}. \quad$ **Ans.**

(*c*) Efficiency $= \dfrac{\text{Work done per kg}}{\text{Energy supplied per kg}}$

$$= \frac{62.28}{\dfrac{V_1^2}{2g}} = \frac{62.28 \times 2 \times 9.81}{35 \times 35} = \mathbf{99.74\%}. \quad \textbf{Ans.}$$