INSTITUTE OF TEXTILE TECHNOLOGY

STRENGTH OF MATERIAL

3rd Semester Mechanical Engineering

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Simple Stress and Strain :-

Stress :- It is defined as Internal resistance force per unit Cross-sectional area.
Mathematically,
Stress is identified by f

$$\boxed{f = \frac{P}{A} = \frac{\text{Force}}{\text{Cross sectional area}}}$$

* Types OF force :-

There are three types of force
a) Tensile force
b) Compressive force
c) Shear force

a) Tensile Force →

It is a such type of force when a body is subjected to it, then it will be Elongated

b) Compressive force →

It is a type of force, when a body is subjected to it, then there will be Contraction

c) shear force or Tangential Force →

, When a body is subjected to it, then there will be angular deformation.

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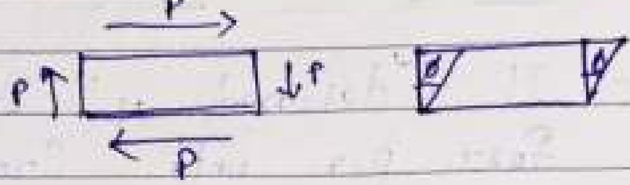

* Types of Stress:-

When the types of force is tencile, then ~~force~~ stress developed in it is tensile in nature

When the nature of force is Compressive, then the stress developed will be Compressive in nature.

When the nature of force is Shear or tangential, then the Stress developed will be shear stress.

* Hooke's law:-

According to Robert Hooke's Stress is proportional to strain, within Elastic limit

Mathematically,

$$\text{stress} \propto \text{strain}$$

$$f \propto e$$

We know, Strain:- It is defined as change in ~~la~~ length to the original length

$$e = \text{Change in length} / \text{original length}$$

$$\boxed{e = \delta l / l}$$

strain is unitless.

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$$\boxed{f = E \cdot e}$$ E SI unit $\rightarrow N/m^2$

This E = Young's Modulus or Modulus of Elasticity

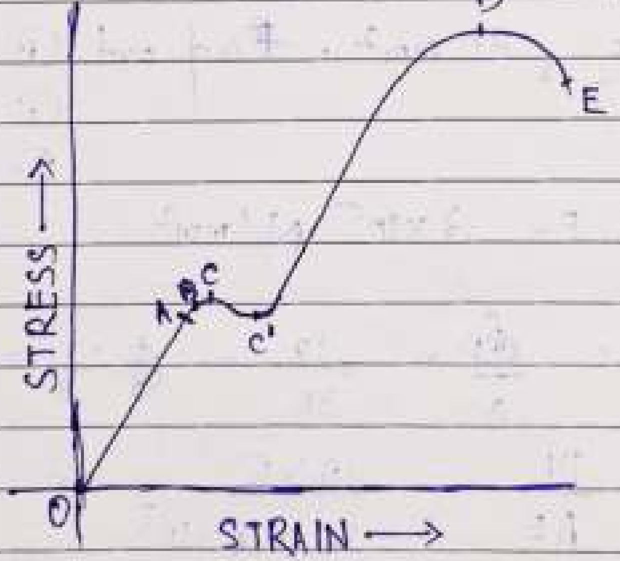
Now, Putting the value of $f = \frac{P}{A}$ and $e = \frac{\delta L}{L}$

Hence, $\frac{P}{A} = E \cdot \frac{\delta L}{L}$

$$\boxed{\delta L = \frac{PL}{AE}}$$

where, $P \rightarrow$ force
$L \rightarrow$ Original length of the body
$A \rightarrow$ cross sectional Area

* Stress v/s strain diagram of M.S. rod Material by using UTM :-



OA $\rightarrow$ Proportional limit
B $\rightarrow$ Elastic limit
C $\rightarrow$ upper Yield Point
C' $\rightarrow$ Lower Yield Point
D $\rightarrow$ ultimate load value
E $\rightarrow$ Fracture or Breaking Point

Ultimate stress refers to the maximum stress that a given material can understand under an applied force.

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Ans → It is Proportional limit, it belongs to within elastic limit i.e stress exactly directly Proportional to strain.

Though,

B → , It is Elastic limit, but it has some Plastic in nature

C → It is called upper yield Point.
yield means softness.

C' → It is called lower yield Point, at which elongation, of the member Take Place, without further, increase of the load

D → It is the Maximum, load value, of a body to with stand Just before fracture.

E → It is the last point known as fracture or rupture. Or Breaking point. It is just after ultimate load value.

Q// A Member of length 5m is subjected to a tensile force of 10N. Its cross sectional area is 30 mm². Find out ① stress
② strain
③ Change in length.

Take $E = 2 \times 10^5 \ N/mm^2$

Sol→ ① $\text{stress} = \frac{P}{A} = \frac{10}{30} = \frac{1}{3} = 0.33 \ N/m^2$

③ $\delta l = \frac{PL}{AE} = \frac{10 \times 5}{30 \times 2 \times 10^5} = 8.33 \times 10^{-6} \ m$

② $e = \frac{\delta l}{l} = \frac{8.33 \times 10^{-6}}{5} = 1.66 \times 10^{-6}$

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Q// A bar of 5cm diameter and 400cm long is acted upon, by a load of 10 tonnes. It is found to Extend 10cm. Find

Find (1) stress
(2) strain
(3) E (Young Modulus)
(4) Workdone = F×S = average force × displacement

Sol→ (1) stress = $\frac{P}{A}$ = Area = $\frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25$
$= \frac{3.14}{4} \times 25$
$= 0.785 \times 25$
$= 19.625$

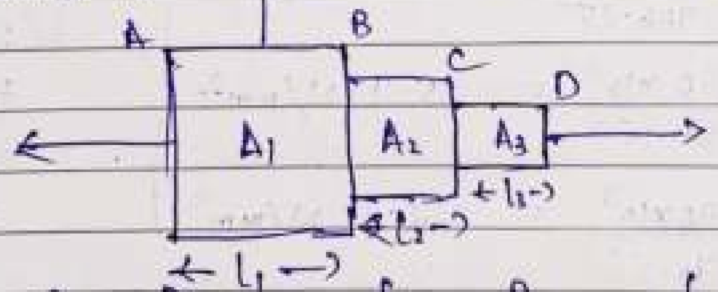
(1) stress $f = \frac{P}{A} = \frac{10000}{19.625} = 509.55$

(2) strain = $e = \frac{\delta L}{L} = \frac{10}{400} = 0.025$

(3) $E = \frac{f}{e} = \frac{509.55}{0.025} = 20382$ N/m²

(4) $W = 5000 \times 10 = 50000$

In, this Condition, the member is not having uniform process as shown below



$f_1 = \frac{P}{A_1}$, $f_2 = \frac{P}{A_2}$, $f_3 = \frac{P}{A_3}$

AB elongation – δL_1
BC → δL_2, CD → δ_3,

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$$\delta l_1 = \frac{Pl_1}{A_1E}, \quad \delta l_2 = \frac{Pl_2}{A_2E}; \quad \delta l_3 = \frac{Pl_3}{A_3E}$$

$$\boxed{\delta l = \delta l_1 + \delta l_2 + \delta l_3}$$

Q11 A round copper rod 550mm long has a diameter of 30mm, over a length of 200mm, a diameter of 20mm over a length of 200mm and a diameter of 10mm over its remaining length. Determine the stress in each section and elongation of rod, when a subjected to force 30kN. Take E = 100 kN/mm²

Sol → $d_1 = 30mm$, $P = 30kN = 30\times10^3 N$, $l_1 = 550mm$
$d_2 = 20mm$, $l_2 = 200mm$
$d_3 = 10mm$, $l_3 = 200mm$

$$Area_1 = \frac{\pi}{4}\times(30)^2 = 706.85\ mm^2$$
$$Area_2 = \frac{\pi}{4}\times(20)^2 = 314.15\ mm^2$$
$$Area_3 = \frac{\pi}{4}\times(10)^2 = 78.53\ mm^2$$

$$\frac{kNm}{mm^2\times\frac{kN}{mm^2}}$$

$$\therefore f_1 = \frac{P}{A_1} = \frac{30\times10^3}{706.85} = 42.44\ N/mm^2$$
$$f_2 = \frac{P}{A_2} = \frac{30\times10^3}{314.15} = 95.49\ N/mm^2$$
$$f_3 = \frac{P}{A_3} = \frac{30\times10^3}{78.53} = 382.01\ N/mm^2$$

$$\therefore \delta l = \delta l_1 + \delta l_2 + \delta l_3 = (233.43 + 190.99 + 764.03)\ m = 1188.45m$$

$$\delta l_1 = \frac{Pl_1}{A_1E} = \frac{30\times10^3\times550}{706.85\times100} = 233.43\ m$$
$$\delta l_2 = \frac{Pl_2}{A_2E} = \frac{30\times10^3\times200}{314.15\times100} = 190.99m$$
$$\delta l_3 = \frac{Pl_3}{A_3E} = \frac{30\times10^3\times200}{78.53\times100} = 764.03\ m$$

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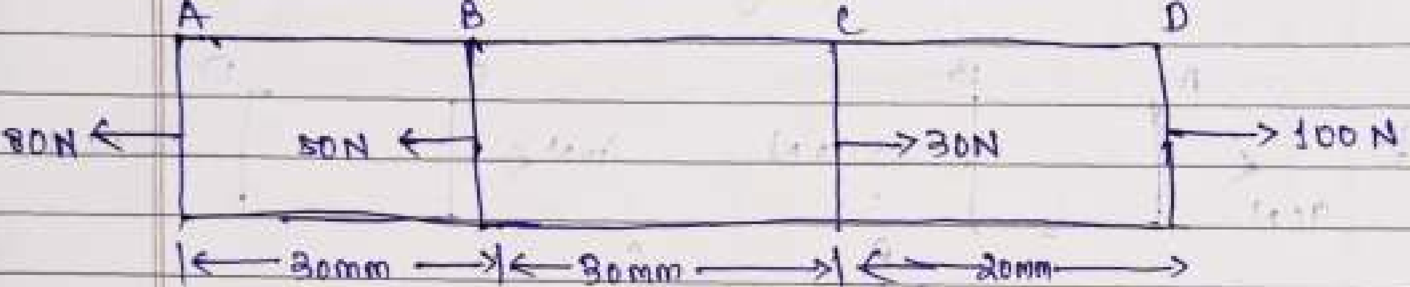
* Principle of Superposition :-

In engineering field application, there are some situation, where a member subjected to, both external as well as internal process.

In, that situation, we have ~~too~~ to split the Section, in various parts according to our Choice and find out net Elongation by summing up Elongation of each Section, this principle is known as Principle of Superposition.



Find out Net Elongation of the member as shown above

Given, $A = 30 mm^2$

$E = 2 \times 10^5 \; N/mm^2$

Sol →

$$\delta l_1 = \frac{P_1 l_1}{AE} = \frac{80 \times 30}{30 \times 2 \times 10^5} = 4 \times 10^{-4} \; mm$$

$$\delta l_2 = \frac{P_2 l_2}{AE} = \frac{130 \times 30}{30 \times 2 \times 10^5} = 6.5 \times 10^{-4} \; mm$$

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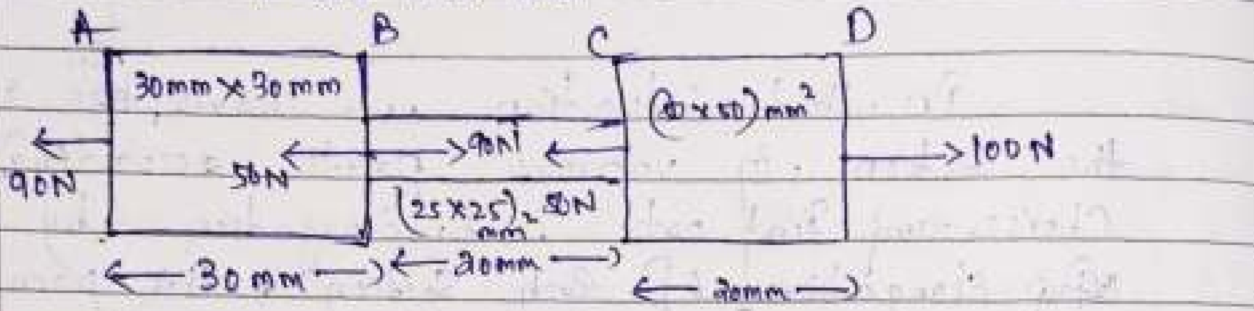
$$\delta l_3 = \frac{P_3 l_3}{AE} = \frac{100 \times 20}{30 \times 2 \times 10^5} = 3.33 \times 10^{-4}\ mm$$

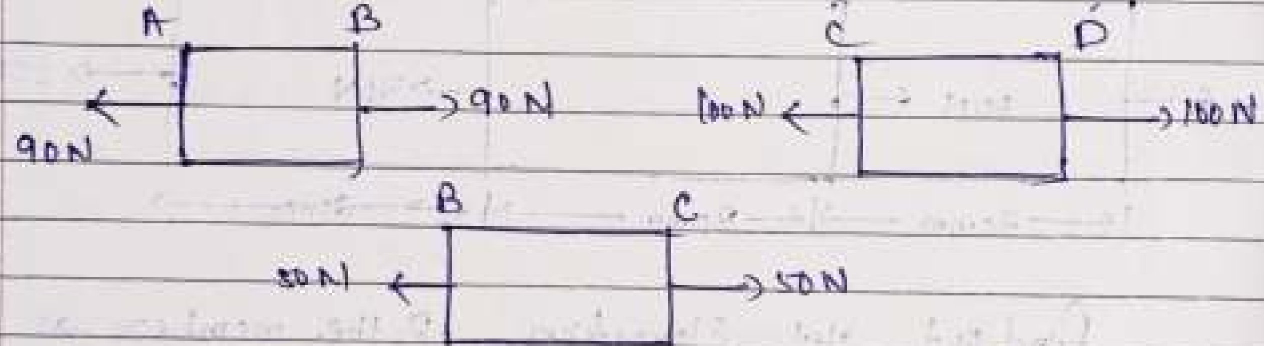
$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$
$$= 1.383 \times 10^{-3}\ mm$$

Q//



Find out Net Elongation, Take $E = 2 \times 10^5\ N/mm^2$

Sol⇒



$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{90 \times 30}{900 \times 2 \times 10^5} = 1.5 \times 10^{-5}\ mm$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{50 \times 20}{625 \times 2 \times 10^5} = 8 \times 10^{-6}\ mm$$

$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{100 \times 20}{2500 \times 2 \times 10^5} = 4 \times 10^{-6}\ mm$$

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 = 1.5 \times 10^{-5} + 8 \times 10^{-6} + 4 \times 10^{-6}$$
$$= 2.7 \times 10^{-5}\ mm$$

of uniform length

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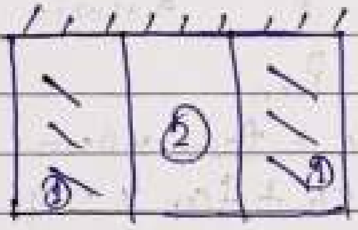
* Stress on Composite Member :-

Composite member is a such type of Member, which is having combination of more than one type of material.

Ex- Aluminium combine with stainless steel so on.

The Composite member is having light weight in nature, but load carrying capacity is two to three times more than conventional Member



Concept-1

$\boxed{\text{Total load applied} = \text{load sharing by member 1} + \text{member 2}}$

$\boxed{\text{Strain on member 1} = \text{Strain on member 2}}$

$\boxed{P = P_1 + P_2}$ — (1)

$\boxed{e_1 = e_2}$ — (2)

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$$\boxed{P = f_1 \times A_1 + f_2 \times A_2} \quad \text{--- (1)}$$

$\Rightarrow e_1 = e_2$

$$\Rightarrow \boxed{\frac{f_1}{E_1} = \frac{f_1}{E_2}} \quad \text{--- (2)}$$

Q// A load of 30 tones is applied on a short Concrete Coulomb of 25cm x 25cm. The Coulomb is reinforced by steel Bars of total area 56cm^2. If, the modulus of Elasticity for steel is 15 times, that of Concrete, (i) Find stress in steel and Concrete. (ii) If, the stresses in Concrete should not Exist 40kg/cm^2, Find the area of steel required so that, colum.

Sol> given, $P = 30{,}000kg$, $A_{colum} = 25cm \times 25cm = 625cm^2$

$A_{steel} = 56\,cm^2$, $E_s = 15\,E_{con}$

$P = P_s + P_{con}$

$30{,}000 = f_s \times A_s + f_{con} \times A_{con}$

$30{,}000 = f_s \times 56 + f_{con} \times 569$ --- (i)

$A_{concrete} = A_{column} - A_{steel}$

$= 625cm^2 - 56cm^2$

$= 569\,cm^2$

~~$e_1 = e_2$~~

~~$\frac{f_1}{E_1} = \frac{f_1}{E_2}$~~

$e_s = e_c$

$\frac{f_s}{E_s} = \frac{f_c}{E_c}$

$\frac{f_s}{15\,E_{con}} = \frac{f_{con}}{E_{con}}$

$15 \times f_{con} = f_s$ --- (ii)

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~~Now~~ putting the value of Eq n(ii) in Eqn (i), we get

$$30000 = \left[15 \times f_{con} \times 56 + f_{con} \times 569\right]$$

$$30000 = f_{con}\left[(15 \times 56) + 569\right]$$

$$30000 = f_{con}\left[840 + 569\right]$$

$$30000 = f_{con} \times 1409$$

$$\therefore f_{con} = \frac{30000}{1409} = 21.29 \ kg/cm^2$$

may, support a load of 60 tones. $\therefore f_s = 15 \times f_{con} = 15 \times 21.29$

$$= 319.35 \ kg/cm^2$$

(ii) given

$f_{Concrete} = 40 \ kg/cm^2$, $E_s = 15 E_c$

$P = 60{,}000 \ kg$

$A_s =$

$A_{column} = 625$

$$P = P_s + P_{con}$$

$$60000 = (f_s \times A_s) + (f_{con} \times A_{con})$$

$$60000 = 600 \times A_s + 40 \times (625 - A_s)$$

$$60000 = 600 A_s + 25000 - 40 A_s$$

$$60000 = 25000 + 560 A_s$$

$$60000 - 25000 = 560 A_s$$

$$35000 = 560 A_s$$

$$\frac{35000}{560} = A_s \Rightarrow 62.5 \ cm^2$$

$$e_s = e_{con}$$

$$\frac{f_s}{E_s} = \frac{f_{con}}{E_{con}}$$

$$\frac{f_s}{E_s} = \frac{40}{E_{con}}$$

$$\frac{f_s}{15 E_{con}} = \frac{40}{E_{con}}$$

$$f_s = 40 \times 15 \ cm^2$$

$$= 600 \ kg/cm^2$$

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$A_{concrete} = A_{column} - A_{steel}$
$= 625 - A_s$

~~Notes~~

* Stresses in Composite member of different length:-

There are two Concept :-

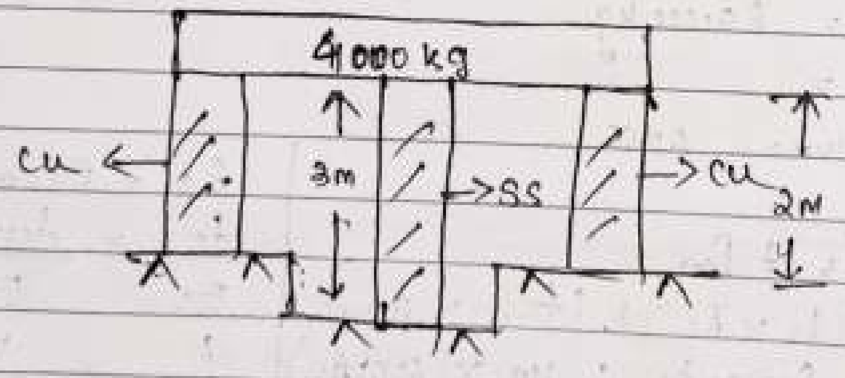
$$\boxed{\begin{array}{l} \text{(i) Total load applied} = \text{load sharing by member (1)} + \\ \quad P = P_1 + P_2 \qquad\qquad + \text{load sharing by member (2)} \end{array}}$$ —— ①

(ii) $\delta l_1 = \delta l_2$ —— ②

$f \propto e$
$f = Ee$
$f = E\frac{\delta l}{l}$

$$\boxed{\delta l = \frac{f\,l}{E}} \text{ —— ②}$$

Q11



Two Copper rod and one steel rod is of 2.5cm diameter together, support a load of 4000 kg as shown in Figure. Find the stresses in Each rod. Take $E_s = 2 \times 10^6$ kg/cm^2
$E_c = 1.1 \times 10^6$ kg/cm^2

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Solution given, $P = 4000\,kg$

$$\boxed{4000 = P_s + 2P_{cu}} \quad \text{——— (i)}$$

$$\boxed{\delta l_s = \delta l_{cu}} \quad \text{——— (ii)}$$

$$4000 = f_s \times A_s + 2 f_{cu} \times A_{cu}$$
$$4000 = \left(f_s \times \frac{\pi}{4}(2.5)^2\right) + \left(2 \times f_{cu} \times \frac{\pi}{4}(2.5)^2\right)$$

$$4000 = 4.9\left(f_s + 2f_{cu}\right)$$

$$\Rightarrow f_s + 2f_{cu} = \frac{4000}{4.9} = 816.32$$

$$\delta l_s = \delta l_{cu}$$

$$\frac{f_s}{E_s} \times l_s = \frac{f_{cu}}{E_{cu}} \times l_{cu}$$

$$\frac{f_s}{2 \times 10^6} \times 3 = \frac{f_{cu}}{1.1 \times 10^6} \times 2$$

$$f_s \times 1.5 \times 10^{-6} = f_{cu} \times 1.81 \times 10^{-6}$$
$$f_s = f_{cu} \times \frac{1.81 \times 10^{-6}}{1.5 \times 10^{-6}}$$
$$f_s = f_{cu}(1.20) \quad \text{——— (iii)}$$

Now, Putting the value of Eqn (iii) we get

$$\Rightarrow f_{cu}(1.20) + 2f_{cu} = 816.32$$
$$\Rightarrow 3.20 f_{cu} = 816.32$$
$$\Rightarrow f_{cu} = \frac{816.32}{3.20} = 255.1 \; kg/m^2$$

$$f_s = f_{cu}(1.20)$$
$$= 255.1 \times 1.20$$
$$= 306.12 \; kg/m^2$$

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Temp Stress :-

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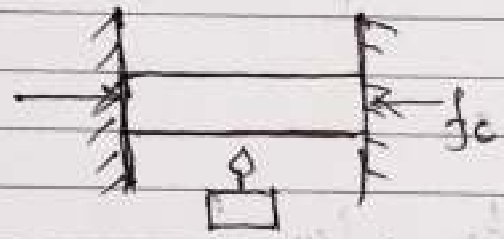
When, a body is subjected to rise in temp., then their must be Expancion.

Similarly, an object, if there is fall in temp., then there must be Contraction, but in both the Cases no stressess, will be developed, because, the ends are free

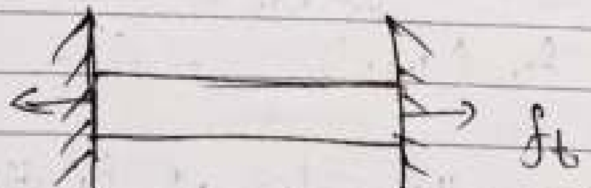
While, both the Ends are rigidly fixed and Expansion or Contraction prevented, then there will be development of stress on the member

Rise in temp. (Heat added) :-



In, this Situation, both the Ends are rigidly fixed and Expansion is prevented so, the stresses developed, Each Compressive in nature and this Stress is known as Thermal stress or temp. stress.

Fall in temp. (Heat Substraction) :-



In, this Situation, there is falling temp. So, Contraction must be there but, it is prevented

So, stresses will be developed which is tensile in nature and it is called temp stress or thermal stress.

According to physical science, during heating and cooling, there is important parameter having major role that is Co-efficient of linear Expansion (α).

(α) - It is defined as change in length to original length or degree rise in temp.

Mathematically, $\alpha = \frac{\delta l}{l} / t^\circ C$

$$\alpha = \frac{\delta l}{lt}$$

$$\boxed{\delta l = \alpha l t}$$

δl = change in length
l = original in length
$t^\circ C$ = rise in or fall in temp.

$\alpha l = \alpha l t$
$\delta l = \alpha l t$
$f = E\alpha t$
$e = \alpha t$

Determination of temp. Stress:-

We know, $f = E \cdot e$

$$f = E \cdot \frac{\delta l}{l}$$

$$f = E \cdot \frac{\alpha l t}{l}$$

$$\boxed{f = E\alpha t} \Rightarrow \text{temp. stress}$$

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Temp^r strain :-

$$e = \frac{\delta l}{l}$$

$$\boxed{e = \frac{\alpha l t}{l} = \alpha t}$$

$\boxed{\text{SI unit of } \alpha \text{ is } 1/°C}$

Q- A rod 2m long, is at temp of 10°C. Find the Expansion, of the rod, if temp is raised to 80°C. If this Expansion is prevented. Find the stress in the material.

Take, $E = 2 \times 10^6 \ kg/cm^2$
$\alpha = 12 \times 10^{-6} /°C$

Sol^n -> Given, $l = 2m$
Rise in $t = 70°C$

$$\delta l = \alpha l t$$
$$= (12 \times 10^{-6}) \times 2 \times 70°C$$
$$= \cancel{2.4 \times 10^{-4}} \ 1.68 \times 10^{-3} \ m$$

$$f = E \alpha t$$
$$= (2 \times 10^6) \times (12 \times 10^{-6}) \times 70$$
$$= 1680 \ \frac{kg}{cm^2} \times \frac{1}{°C} \times °C$$
$$= 1680 \ kg/cm^2$$

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Q/ A steel 1m long is fixed at Ends and subjected to a pool of 9kN, determine the residual stress. Determine, the temp in 20°C. Diameter of rod is 12 mm.

$E = 200\, kN/mm^2$

$\alpha = 60 \times 10^{-6} /°C$

Sol^n -> given, $l = 1m$

$P = 9kN = 9000\,N$

$D = 12\,mm$

$t = 20°C$

Case - 1

$$f_t = \frac{P_t}{A_t} = \frac{9000}{\frac{\pi}{4}(12)^2} = 0.07\ kN/mm^2 \text{ (tensile)}$$

Case - II Rise in temp.

$$f = E\alpha t$$
$$= 200 \times (60 \times 10^{-6}) \times 20$$
$$= 0.064\ kN/mm^2$$

Residual stress —

$$0.07 - 0.064 = 6 \times 10^{-3}\ kN/mm^2$$

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Temp stress in Composite Member

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For composite member, behaves as a single member

one is having more value of α and other one is less value of α

We know if α value is more then Expansion and contraction will be more. similarly, For less value of α Expansion and contraction will be less.

There are two Concept, For numerical Point of view

(i) Compressive force of member 1 is Equal to tensile force on Member 2

(ii) Strain on Member 1 + Strain on Member 2 is Equal to rise in temp ($\alpha_{mem1} - \alpha_{mem2}$)

Mathematically

$f = \frac{P}{A}$

(i) $\boxed{f_c \times A_c = f_t \times A_t}$ $P = f \times A$

$f = E \cdot e$

(ii) $\boxed{e_1 + e_2 = T(\alpha_1 - \alpha_2)}$ $e = \frac{f}{E}$

Q-1) A 2.2 cm Copper rod Passes Centrally through a Steel tube of 4cm internal diameter and 5cm external diameter, while at 28°C, the Ends are rigidly Fastened. Find the intensity of Stress in each Metal, if heated to 128°C.

Take $E_s = 2 \times 10^5 \ N/mm^2$

$E_c = 1.2 \times 10^5 \ N/mm^2$

$\alpha_s = 12 \times 10^{-6} /°C$

$\alpha_{cu} = 18 \times 10^{-6} /°C$

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Sol^n->

$f_{cu} \times A_{cu} = f_s \times A_s$ ——— (i)

$e_s + e_{cu} = 100 \, (\alpha_{cu} - \alpha_s)$ ——— (ii)

$A_{cu} = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(2.2)^2 \, cm^2$

$= 3.80 \, cm^2$

$A_s = \frac{\pi}{4}(d_0^2 - d_i^2)$

$= \frac{\pi}{4}[(5)^2 - (4)^2]$

$= \frac{\pi}{4}[25 - 16]$

$= \frac{\pi}{4} \times 9 = 7.06 \, cm^2$

$\rightarrow f_{cu} \times 3.8 = f_s \times 7.06$

$\rightarrow f_{cu} = f_s \times \frac{7.06}{3.8}$

$\rightarrow \boxed{f_{cu} = 1.8 f_s}$ ——— (iii)

$e_s + e_{cu} = 100 \, (\alpha_{cu} - \alpha_s)$

$\frac{f_s}{E_s} + \frac{f_{cu}}{E_{cu}} = 100 \, (6 \times 10^{-6})$

$\frac{f_s}{E_s} + \frac{1.8 f_s}{E_{cu}} = 6 \times 10^{-4}$

$f_s \left(\frac{1}{E_s} + \frac{1.8}{E_{cu}}\right) = 6 \times 10^{-4}$

$f_s \left(\frac{1}{2 \times 10^5} + \frac{1.8}{1.2 \times 10^5}\right) = 6 \times 10^{-4}$

$f_s \times (2 \times 10^{-5}) = 6 \times 10^{-4}$

$f_s = \frac{6 \times 10^{-4}}{2 \times 10^{-5}} = 30$

Now, Putting the value of f_s in Eq^n (iii), we get

$f_{cu} = 1.8 \times f_s$

$= 1.8 \times 30$

$= 54 \, N/mm^2$

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Elastic Constant

There are three types of Elastic Constant

N/mm^2 (unit)

E = Young modulus = ~~simple stress/simple~~ strain

K = Bulk modulus = N/mm^2 (unit)

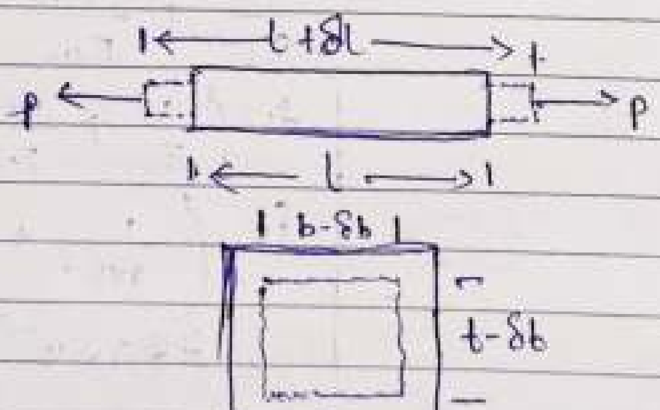
G = Rigidity modulus = N/mm^2 (unit)

E is defined as simple stress by simple strain.

K is defined as ratio of Normal stress to volumetric strain

G is defined as ratio of shear stress to shear strain

For, a rectangular block, when it is Subjected to a tensile Force, we know its length will be increase, at the same time, its breadth will be decrease and thickness also decrease as shown below.



Let original length = L   Final length = L+δL

original breadth = b   Final breadth = b-δb

original thickness = t   Final thickness = t-δt

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Dues to this Effect, there are two types of strain developed on the member

1. longitudinal strain
2. Lateral strain

Longitudinal strain developed along the longitudinal axis and lateral strain develop along the transverse axis.

Poisson's ratio :- $\left(\mu \text{ or } \frac{1}{m} \right)$ (0.25 → 0.33)

It is defined as, within Elastic limit, lateral strain to longitudinal strain bears a constant ratio known as Poisson's ratio

Mathematically, μ or $\frac{1}{m} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

longitudinal strain $= \frac{\delta l}{l}$

lateral strain $= \frac{\delta b}{b}$ and $\frac{\delta t}{t}$ and $\frac{\delta d}{d}$

Q→ A bar of steel 28 ~~mm~~ cm in diameter was subjected to a tensile load of 6 tone and the measured extension on a 20 cm gaze length was 0.01 cm and change in diameter was 0.0~~30 cm~~ 0088 cm. Calculate Poisson's ratio and the E.

Sol→ $d = 28cm$ $= \frac{\pi}{4}(28)^2 = 615.44$

$P = 6 = 6000\,kg$

$l = 20cm$

$\delta l = 0.01cm$

$\delta d = 0.0088cm$

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lateral strain = $\frac{\delta d}{d} = \frac{0.00038}{28} = 1.35 \times 10^{-5}$

longitudinal strain = $\frac{\delta L}{L} = \frac{0.01}{20} = 5 \times 10^{-4}$

$\therefore \mu = \frac{1.35 \times 10^{-5}}{5 \times 10^{-3}} = \cancel{3.375 \times 10^{-3}}\ 0.27$

$f = E \times e \qquad \delta l = \frac{PL}{AE}$

$\therefore E = \frac{f}{e}$

$E = \frac{P/A}{\frac{\delta L}{L}} = \frac{\frac{3000}{615.44}}{5 \times 10^{-4}} = \frac{9.74}{5 \times 10^{-4}} = \cancel{19940}\ 1949820$

Q) A steel bar 3m long (3000 mm), 30 mm wide, 15 mm thick is subjected to a pool of 30kN, in the dir^n of its length. Find δl, δb, δthickness.

$E = 2 \times 10^5 \ N/mm^2$

$\mu = 0.35$

Sol^n) $l = 3m = 3000 mm$

$b = 30 mm$

$h = 15 mm$, Cross sectional area = $b \times t$ $= 30 \times 15$ $= 450 mm^2$

$P = 30 kN = 30000 N$

$\delta l = \frac{30000 \times 3000}{450 \times 2 \times 10^5} = \frac{PL}{AE}$

$= \frac{90000000}{90000000} = 1\ mm$

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$$\frac{1}{m} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$0.35 = \frac{\text{lateral strain}}{\frac{1}{3000}}$$

$$\text{lateral strain} = 0.35 \times \frac{1}{3000}$$

$$\text{lateral strain} = 1.16 \times 10^{-4}$$

$$\text{lateral strain} = \frac{\delta b}{b} = 30$$

$$1.16 \times 10^{-4} \times 300 = \delta b$$

$$\boxed{3.48 \times 10^{-3} = \delta b}$$

$$\text{lateral strain} = \frac{\delta t}{t}$$

$$1.16 \times 10^{-4} \times 15 = \delta t$$

$$\boxed{1.74 \times 10^{-3} = \delta t}$$

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* Volumetric strain of a rectangular bar

* Volumetric Strain

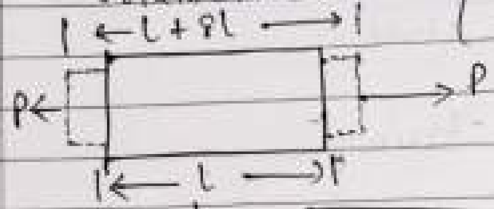

Fig-2

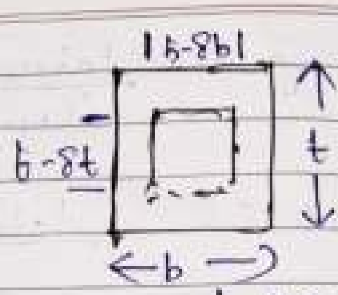

Fig-3

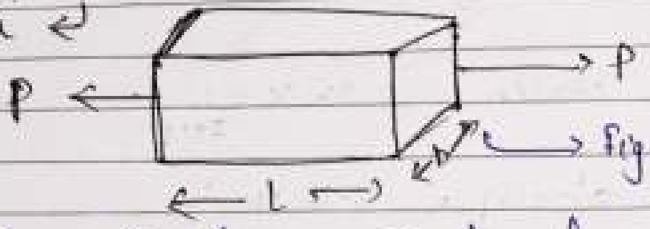

Fig-1

Let us Consider a rectangular block
original length = l
" breadth = b
" thickness = t
and it is Subjected to a tensile Force of P

Final length of the member will be $= l+\delta l$
Final breadth of the member will be $= b-\delta b$
Final thickness of the member will be $= t-\delta t$

We know original volume of the member
$= (l \times b \times t)$
and we know Final volume of the member
$= (l+\delta l) \times (b-\delta b) \times (t-\delta t)$
$= lbt + bt\delta l - lb\delta t - lt\delta b$ (neglecting other smaller values)

so, $\boxed{\text{Volume strain } (e_v) = \dfrac{\text{Change in volume}}{\text{original volume}}}$

$= \dfrac{\text{Final volume} - \text{Initial volume}}{\text{original volume}}$

~~∴ Final volume~~

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$$e_v = \frac{\cancel{lbt} + bt\delta l - lb\delta t - lt\delta b - \cancel{lbt}}{lbt}$$

$$= \frac{bt\delta l}{lbt} - \frac{lb\delta t}{lbt} - \frac{lt\delta b}{lbt}$$

$$= \frac{\delta l}{l} - \frac{\delta t}{t} - \frac{\delta b}{b}$$

$$= \frac{\delta l}{l} - 2\frac{\delta t}{t}$$

$e_v = \frac{\delta v}{v}$ = longitudinal strain − 2 lateral strain

we know, $\frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{1}{m}$ or μ

$$\boxed{\therefore \text{lateral strain} = \frac{1}{m} \times \text{longitudinal strain}}$$

$e_v = \frac{\delta v}{v}$ = longitudinal strain $- 2\left[\frac{1}{m} \times \text{longitudinal strain}\right]$

= longitudinal strain $\left[1 - 2 \times \frac{1}{m}\right]$

$$\boxed{e_v = \frac{\delta v}{v} = e\left[1 - \frac{2}{m}\right]}$$

where δv = Change in volume

v = original volume

$1/m$ = Poisson's ratio

Imp $\frac{\delta L}{L} = e$] longitudinal strain

$\frac{\delta x}{x}, \frac{\delta d}{d}, \frac{\delta b}{b}, \frac{\delta t}{\delta t}$] lateral strain

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Q) A vertical circular bar 20mm dia, 4m long carries, a tensile load of 40kN. Calculate

(i) Elongation

(ii) Decrease in dia. (δd)

(iii) volumetric strain

IF Poisson's ratio = 0.3

$E = 2\times10^5\ N/mm^2$

Sol->

$P = 40kN = 40\times10^3 N$, Area $= \frac{\pi}{4}(d^2)$

$L = 4m$

$d = 20mm$ $= \cancel{314 mm^2}\ 314.15\ mm^2$

$\therefore$ (i) $\delta l = \frac{PL}{AE} = \frac{40\times10^3 \times 4000}{314\times 2\times10^5}$

$= \frac{160000}{62800000} = 2.54\times10^{-3} mm$

$\mu =$ $\frac{\text{lateral strain}}{\text{longitudinal strain}}$

$0.3 = \frac{\delta d/d}{\delta L/L}$

$0.3 = \frac{\delta d/20}{(2.54/4000)}$

$0.3 = \frac{\delta d/20}{6.35\times10^{-4}}$ $e = \frac{\delta L}{L}$

$20\times0.3\times6.35\times10^{-4} = \delta d$

$\delta d = 3.81\times10^{-3}\ mm$

$e_v = 6.35\times10^{-4}[1-2\times0.3]$

$= 2.54\times10^{-4}$

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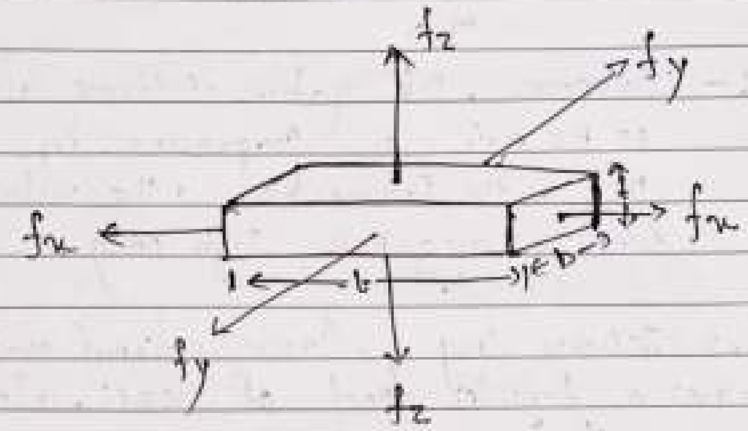
* Volumetric Strain of a rectangular block, When it is subjected to, three mutually perpendicular Stresses (i.e. along x-x, y-y, z-z axes)



In, this situation, there are three stresses, all are tensile in nature.

When, we will along x-axis, the stress fx will be elongative in nature so, nature of strain is longitudinal and it is called longitudinal strain. At the same time the effect along y-axis and z-axis, will be compressive in nature.

So, the nature of strain, along Y and Z axis is lateral and it is called lateral strain. Similarly, when we will considered the stress, along y-axis is ~~longitudinal~~ tensile in nature, then the strain along it will be longitudinal strain and at the same time the strain x and z axis, will be compressive in nature, so, strain is lateral strain. and so on.

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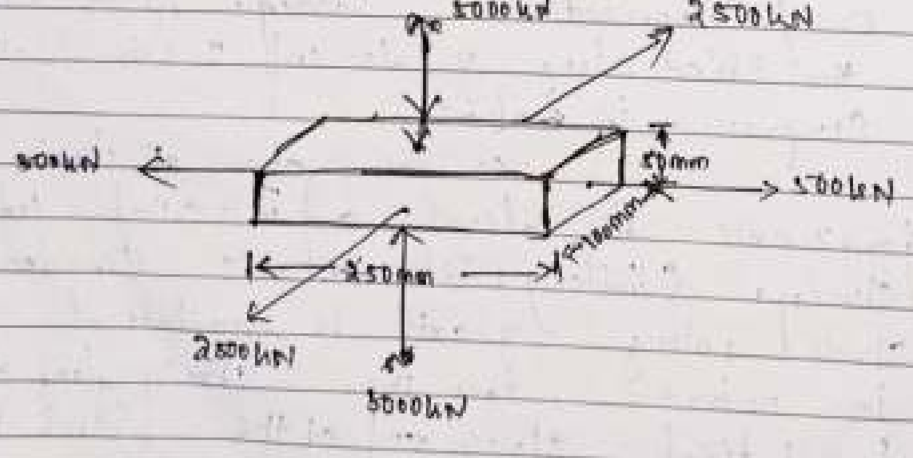
$$\boxed{e_v = \frac{\delta v}{v} = \frac{1}{E}\left[f_x + f_y + f_z\right]\left[1 - \frac{2}{m}\right]}$$

Where, δv = Change in volume
v = Original volume
$\frac{1}{m}$ = Poisson's ratio

Note:- Suppose, the nature of stress along any one of the axis is compressive in nature, then in the formulae, the value of stress will be negative sign only.

Q) A bar 250 mm long, cross-sectional area 100mm × 50mm, carries a tensile load of $\boxed{500kN}$, along length wise, a compressive load of 5000kN, on its 100 mm × 250mm faces and a tensile load of 2500kN, on its, 50mm × 250mm faces. Calculate, the
(i) Change in volume
(ii) volumetric strain
Take $E = 1.8 \times 2 10^5$ N/mm^2
$\frac{1}{m} = 0.25$

Sol^n>



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Let , $P_x = 500 kN$, $f_x = \frac{P_x}{A_x} = \frac{500\times10^3}{5000}$ 100 kN/mm²

$P_y = 2500 kN$, $f_y = \frac{P_y}{A_y} = \frac{2500\times10^3}{250\times50}$ 200 kN/mm²

$P_z = 5000 kN$, $f_z = \frac{P_z}{A_z}$ $\frac{5000\times10^3}{25000} = \frac{200}{1}$ ~~0.2~~ kN/mm²

$A_x = 100 \times 50$

$A_y = 250 \times 50$

$A_z = 250 \times 100$

(i) ~~$e_v = \frac{\delta v}{v}$~~

~~(ii) (i) $e_v = \frac{1}{1.8\times10^5}[1+0.4-0.2][1-2\times0.25]$~~

~~$= \frac{1}{5.55\times10^{-6}}[1.2][0.5]$~~

~~$= \frac{0.6}{5.55\times10^{-6}} = 108108.10$~~

$v = l\times b\times h$

~~(ii) $\delta v = 46.75\times10^{14}$~~

(i) $e_v = \frac{1}{1.8\times10^5}[100+200-200][1-2\times0.25]$

$= \frac{1}{1.8\times10^5}[100][0.5]$

$= 2.77\times10^{-4}$

(ii) $e_v = \frac{\delta v}{v}$, volume $= l\times b\times h$

$= 250\times100\times50$

$= 12,50,000 \text{ mm}^3$

$2.77\times10^{-4} = \frac{\delta v}{v}$

$\therefore \delta v = 2.77\times10^{-4}\times v$

$= 2.77\times10^{-4}\times 12,50,000$

$= 346.25 \text{ mm}^3$

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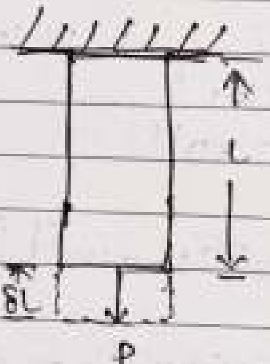
* Strain Energy or Resilience :-

It is a Condition of a body, when it is subjected to a load, the body will Elongate upto a pattern limit, which is within Elastic limit, we know, a body is consist of intermolecular attractive Force, in this situation, there is a External Workdone takes place. At the same time, All stored inside the body.

When, the Force remove away, the body will come back to its original Position by springing that.



According, to the definition, the External work done is Equal strain Energy stored inside the member. So, resilience means, the strain Energy, which is stored inside the member

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Mathematically, determination of Strain Energy :-

External Workdone = Strain Energy stored

Strain energy stored = Force × displacement

$= F \times \delta$

Strain Energy stored = avg. force × displacement

$U = \frac{P}{2} \times \delta L$

A = Cross section area $= b \times \cancel{t}\ lbt$

$V = lbt$

We know,

$f \propto e$

$f = Ee$

$f = E \cdot \frac{\delta L}{L}$

$\therefore \boxed{\delta L = \frac{fL}{E}}$

$\therefore U = \frac{P}{2} \times \frac{fL}{E}$

We know, $f = \frac{P}{A}$ $\therefore \boxed{P = f \times A}$

$U = \frac{f \times A \times f \times L}{2E}$

$= \frac{f^2 V}{2E}$

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There are 3 types of load

(i) Gradually applied load
(ii) Suddenly applied load
(iii) Impact applied load

(i) Gradually applied load :—
In this load, the load will be subjected to a member smoothly.

(ii) Suddenly applied load :—
In this load, the load will be subjected instant/ suddenly. So, its effect will be very severe and mathematically describe below.

In this situation, the load value initially P and finally also P.

$$\text{Strain Energy} = \text{External workdone}$$
$$U = \text{Avg. Force} \times \text{displacement}$$
$$= \left(\frac{P+P}{2}\right) \times \delta L$$
$$\frac{f^2 V}{2E} = \frac{\cancel{2}P}{\cancel{2}} \times \delta L$$
$$\frac{f^2 \times A \times L}{2 \times E} = P \times \delta L$$
$$\frac{f^2 \times A \times L}{2 \times E} = P \times \frac{f}{E} \times L$$
$$f = \frac{2P}{A}$$

Note → For gradually applied load = $f = \frac{P}{A}$

For suddenly applied load = $f = \frac{2P}{A}$

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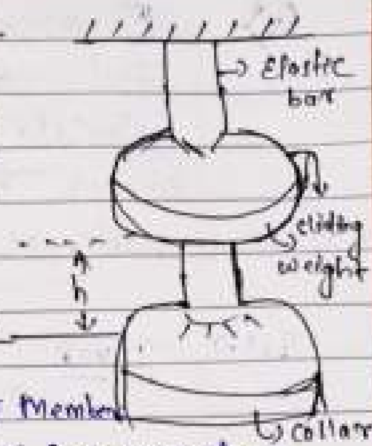
(3) Impact load :-

Let us consider an Elastic bar of original length 'L'. Their is a sliding weight From Clear height 'h'. Striking a Collar which is rigidly attached at the lower End of the bar.



Where, f = Stress | A = Cross section area

E = Young modulus | e = Strain, V = V of Member

Already, we know, Strain Energy Stored = Ext. work done

$$\frac{f^2 V}{2E} = P \times (h + \delta L)$$

$$\frac{f^2 \times A \times L}{2 \times E} = P \times \left(h + \frac{f}{E} L\right)$$

$$\frac{f^2 \times A \times L}{2 \times E} = Ph + \frac{PfL}{E}$$

Multiplying both sides $\left(E/AL\right)$

$$\frac{E}{AL}\left(\frac{f^2 \times A \times L}{2E}\right) = \left(Ph + \frac{PfL}{E}\right)\frac{E}{AL}$$

$$\frac{f^2}{2} = \frac{PhE}{AL} + \frac{PfLE}{EAL}$$

$$\Rightarrow \frac{f^2}{2} = \frac{PhE}{AL} + \frac{Pf}{A}$$

$$\Rightarrow \frac{f^2}{2} - \frac{PhE}{AL} - \frac{P}{A} f = 0$$

$$\Rightarrow \frac{1}{2} f^2 - \frac{P}{A} f - \frac{PhE}{AL} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By comparing it with $ax^2 + bx + c = 0$

$$a = 1/2,\ b = -P/A,\ c = -\frac{PhE}{AL}$$

$$f = \frac{-\left(\frac{-P}{A}\right) \pm \sqrt{\left(\frac{-P}{A}\right)^2 - 4 \times \frac{1}{2} \times \left(-\frac{PhE}{AL}\right)}}{2 \times 1/2}$$

$$= \frac{P}{A} \pm \sqrt{\frac{P^2}{A^2} + \frac{4}{2} \times \frac{PhE}{AL}}$$

$$= \frac{P}{A} \pm \sqrt{\frac{P^2}{A^2} + \frac{2AhE}{PL} \times \frac{P^2}{A^2}}$$

$$= \frac{P}{A} \pm \sqrt{\frac{P^2}{A^2}\left(1 + \frac{2AhE}{PL}\right)}$$

$$= \frac{P}{A} \pm \frac{P}{A}\sqrt{1 + \frac{2AEh}{PL}}$$

$$f = \frac{P}{A}\left[1 \pm \sqrt{1 + \frac{2AEh}{PL}}\right]$$

$$f = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2AEh}{PL}}\right]$$

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Q) An axial pull of 10kN is suddenly applied on a steel rod of 500mm length and 8mm diameter. Calculate the elongation of the rod and the absorb strain energy. Also find the modulus of resilience. Take $E = 2\times10^5\ N/mm^2$.

Sol) given

$P = 10kN$

$l = 500mm$

$d = 8mm$, $E = 2\times10^5\ N/mm^2$

Find out $\delta l = ?$ $\quad V = A\times L$

$U = ?$

Modulus of resilience = ?

$$Area = \frac{\pi}{4}\times(8)^2 = 50.26\ mm^2$$

$$\delta l = \frac{PL}{AE} = \frac{10\times1000\times500}{50.26\times2\times10^5} = \frac{5000000}{10052000} = 0.49\ mm$$ Sol^n →

$$f = \frac{2P}{A} = \frac{2\times10^4}{50.26} = 397.93\ N/mm^2$$

~~9947.47×8N~~

(strain Energy) $U = \frac{f^2 V}{2E}$

$$U = \frac{f^2\times A\times L}{2\times E} \quad \left[\frac{\left(\frac{N}{mm^2}\right)^2\times mm^2\times mm}{N/mm^2}\right]$$

$$= \frac{(397.93)^2\times50.26\times500}{2\times2\times10^5} = \frac{9999980.9}{400000} = \cancel{24.99}$$ 9948.29 N·mm (Ans)

For suddenly applied load $U = U_p$

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Modulus of Resilience (MR) = $\frac{\text{Proof resilience}}{\text{volume}}$

$\frac{U_P}{V} = \frac{9948.23}{50.26 \times 500}$

$= 0.39\ N/mm^2$

$= \frac{U_P}{V} = \frac{24.99}{50.26 \times 10}$

$= 0.04$ (Ans)

Q) A solid steel rod of length 1m, diameter 20mm hangs ~~theme~~, vertically and has a polar ~~seen~~ rigidly attached, at the lower end. Find the maximum stress, induced, when weight of 20kg falls on the polar from a clear height of 150mm.
Take $E = 2 \times 10^5\ N/mm^2$.
Also, find the absorb strain energy, modulus of resilience and maximum instantaneous elongation of the bar.

Solⁿ →

$l = \cancel{1m} = 1000\ mm$
$d = 20mm$
$h = 150mm$
$E = 2 \times 10^5\ N/mm^2$
$P = 20kg$

The load given 20kg means it is represent mass. Actually load = W = P = mg
$= 20 \times 9.81$
$= 196.2\ N$

$\delta l = ?$
$U = ?$

Area $= \frac{\pi}{4} \times (20)^2 = 314.15\ mm^2$

$\delta l = \frac{PL}{AE} = \frac{196.2 \times 1000}{314.15 \times 2 \times 10^5} = \frac{196200}{62830000} = 3.12 \times 10^{-3}$

$U = \frac{f^2 V}{2E} = \frac{f^2 \times A \times L}{2E} = \frac{(194.20)^2 \times 314.15 \times 1000}{2 \times 2 \times 10^5}$

$= 29619.35\ N\text{-}mm$

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induced stress $(f) = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2AEh}{PL}}\right]$

$= \frac{196.2}{314.15}\left[1 + \sqrt{1 + \frac{2 \times 314.15 \times 2\times10^5 \times 150}{196.2 \times 1000}}\right]$

$= \frac{196.2}{314.15}\left[1 + 309.95\right]$

$= \frac{196.2}{314.15} \times 310.95$

$= 194.20\ N/mm^2$

Modulus of resilience $= \frac{U}{V} = \frac{29619.35}{314.15 \times 1000}\ N/mm^2$

Q) A uniform metal bar of rectangular section (40×20)mm is of length 1.5m. Find the strain energy stored in the bar, when a load of 100kN is gradually applied to it. If the elastic limit of the metal with which the bar is made is 160 N/mm². What is the proof resilience and M.R. Take $E = 2\times10^5\ N/mm^2$.

Sol.>

Area $= (40\times20)\ mm = 800\ mm^2$

$l = 1.5m = 1500\ mm$

$P = 100kN = 100\times10^3 N$

$f_E = 160\ N/mm^2$

$V = A\times l = 800\times1500 = 1200000$

$\therefore f = \frac{P}{A} = \frac{100\times10^3}{800} = 125\ N/mm^2$

(strain energy stored) $U = \frac{f^2 V}{2E} = \frac{(125)^2 \times 1200000}{2\times2\times10^5}$

$= 46875\ N\text{-}mm$

Proof resilience

$U_P = \frac{f_E^2 V}{2E}$

$= \frac{(160)^2 \times 1200000}{2\times2\times10^5} = 76800\ N\text{-}mm$

$M.R. = \frac{U_P}{V}$

$= \frac{76800}{1200000} = 0.064\ N/mm^2$

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Saathi

Date __/__/__ **Shear force and Bending Moment of Beam**

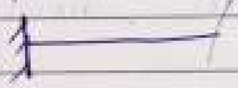
Beam is a structural member ~~with~~ in horizont-el with the Support of Column and column is a vertical member.

Types of Beam :-

(1) Cantilever Beam
(2) Simply supported Beam
(3) Continuous Beam
(4) Fixed Beam
(5) Proped Cantilever Beam
(6) Over hanging beam

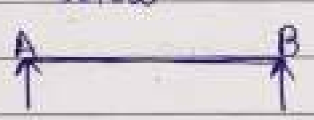
(1) Cantilever Beam :-

It is a type of beam, whose one End is Fixed and other is Free



(2) Simply Supported Beam :-

It is a type of beam, whose both Ends are supported by vertical support (column) as shown below



(3) Continuous beam :-

It is a type of beam, where beam is placed horizontally and there are more than two Supports as shown below

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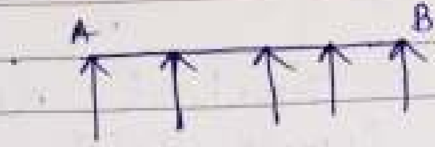


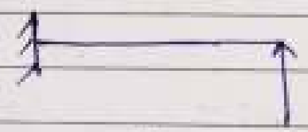
(4) Fixed Beam :-

It is a type of beam, whose both Ends are rigidly Fixed as shown below

Here, at A & B, the Support is rigidly fixed with the Structure.

(5) Proped Cantilever beam :-

It is a type of beam, whose one End is fixed and other End is Supported by a vertical support as shown below



(6) Over hanging beam

(i) Over hanging one End beam :-

It is Condition, in which, the beam is Extended by Some length at one End as shown ~~above~~ below

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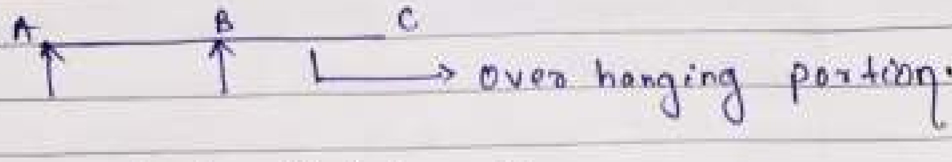
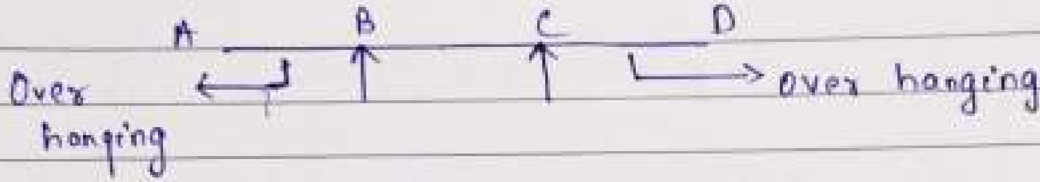


(ii) Both hanging both End beam :-

It is such condition, in which the beam is Extended by suttern length at both ends



* Types of load :-

(1) Point load or Concentrated load
(2) Uniformly distributed load
(3) Gradually varying load

(1) Point load or Concentrated load :-

For Calculation point of view, it is assumed, the load is Exactly acting at a point, so, it is Called Point load.

(2) Uniformly distributed load :-

It is a type of load, which is acting on a beam and distributed uniformly through out the beam as shown below.

1 m = 100 N

2 m = 200 N

10 m = 100 × 10 = 1000 N

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Shear Force :-

It is a Force Subjected on a beam due to which a portion of the beam will shear(slide) with respect to another portion. It is purely the natical, the value of Shear Force about a Section either right of the section or left of the section will be same.

So, shear force is algebraic sum of all the Forces, either right of a section or left of a section will be Equal in Magnitude.

Bending Moment :-

Bending moment of a beam is algebraic sum of moment of all the Forces about a Section either to the left or to the right

* A Cantilever beam with a Point load. Draw S.F. & B.M. Diagram

For shear force diagram :- (~~Point load~~)

-> For Point load, the diagram will be rectangular in nature.
-> For U.D.L, Shear force diagram will be inclined st. line

For Bending moment diagram :-

-> For point load, diagram will be inclined st. line
-> For UDL, the diagram will be Parabolic
-> For Positive value, the diagram will be above the base line and for negative value, the diagram will be below, the base line.

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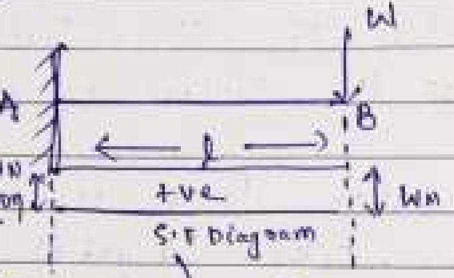
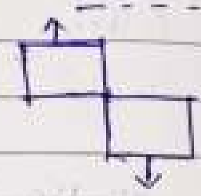
→ For +ve value, the diagram will be above the reference

→ For -ve value, the diagram will be below the reference.

Ex



Sign Convention for SF



+V SF

SF Calculation

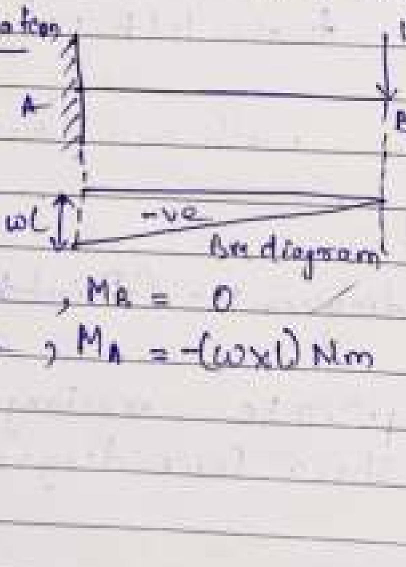
S.F at B (Just right of B) - $F_B = 0$

S.F at B (Just left of B) - $F_B = +WN$

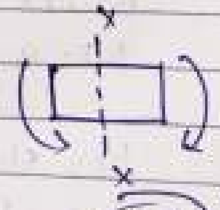
S.F at A - $F_A = +WN$

BM Calculation

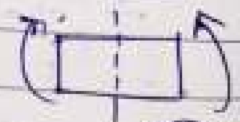
Ex



Sign Convention for BM



-ve BM



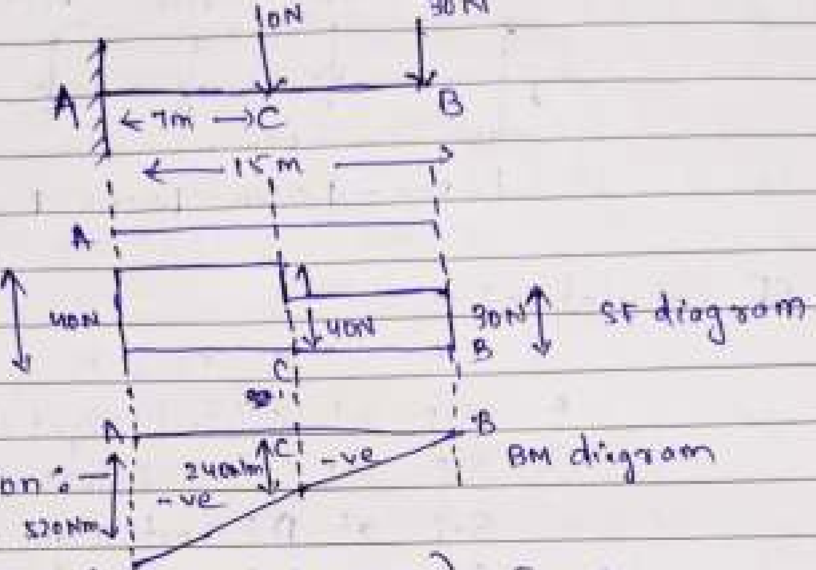
+ve BM

BM at B, $M_B = 0$

BM at A, $M_A = -(W \times l)$ Nm

Date __/__/____

Q7. Draw S.F and BM diagram of a Cantilever beam as shown below



SF Calculation :-

S.F at B (Just right of B) - $F_B = 0$

S.F at B (Just ~~right~~ left of B) - $F_B = +30$ N.

S.F at C (Just right of C) - $F_C = +30$ N

S.F at C (Just left of C) - $F_C = (30+10) = +40$ N

S.F at A - $F_A = (30+10) = +40$ N

BM Calculation :-

BM at B, $M_B = 0$ Nm

B.M at C, $M_C = -(30 \times 8) - (10 \times 0)$

$= -240$ Nm

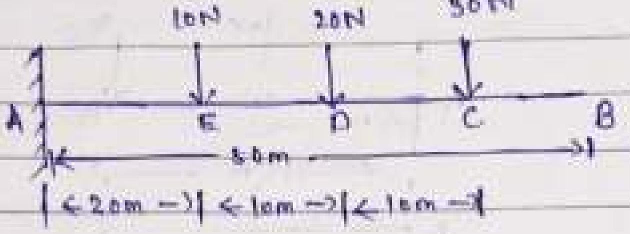
BM at A, $M_A = -(30 \times 15) - (10 \times 7)$

$= -450 - 70$

$= -520$ Nm

Date ___/___/______

Q) Draw S.F and BM diagram of a Cantilever beam as shown below :-



SF Calculation :-

S.F at B, $F_B = 0$

S.F at C (Just right of C), $F_C = 0$

S.F at C (Just left of C), $F_C = 30N$

S.F at D (Just right of D), $F_D = 30N$

S.F at D (Just left of D), $F_D = 50N$

S.F at E (Just right of E), $F_E = 50N$

S.F at E (Just left of E), $F_E = 60N$

BM Calculation :—

BM at B, $M_B = 0$

BM at C, $M_C = 0$

BM at D, $M_D = -(30 \times 10) - (20 \times 0)$

$= -300$ Nm

BM at E, $M_E = -(30 \times 20) - (20 \times 10) - (10 \times 0)$

$= -600 - 200 = -800$ Nm

BM at A, $M_A = -(30 \times 40) - (20 \times 30) - (10 \times 20)$

$= -1200 - 600 - 200$

$= -1800 - 200$

$= -2000$

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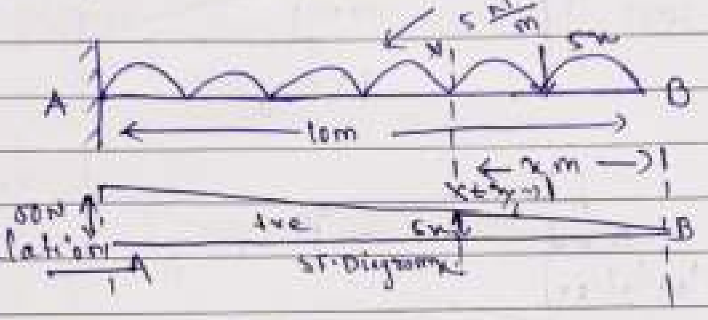
* Cantilever with UDL :-

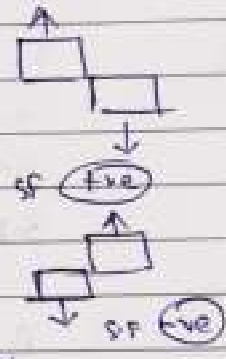
Let us Concider a beam of lm Subjected into udl of wN/m

Assuming a line x-x, at distance of xm From B. , so, SF at x-x = F_x

Q/



S.F calculation



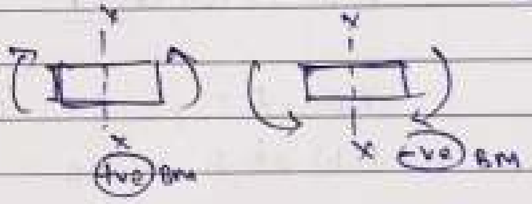
S.F at B, $F_B = 0$

S.F at x, $F_x = +5x$ N $= +5x$N

S.F at A, $F_A = +(10 \times 5)$N

$= +50$N

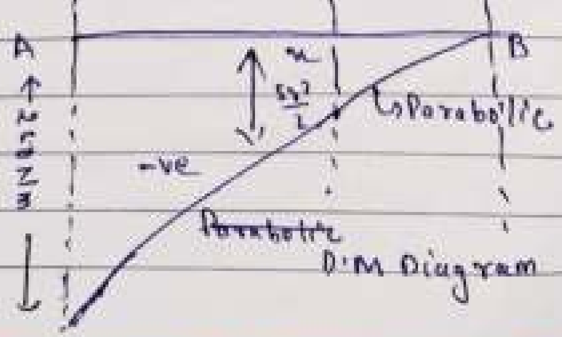
BM Calculation :-



B.M at B, $M_B = 0$N

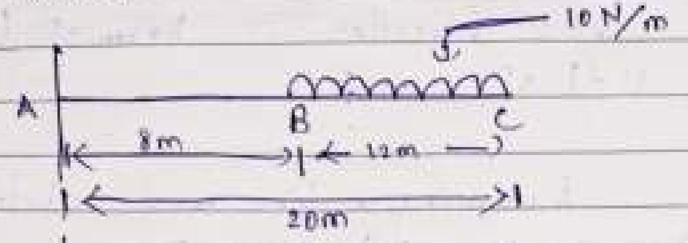
BM at xx, $M_x = (5x) \times \left(\frac{x}{2}\right) = -\frac{5x^2}{2}$ Nm

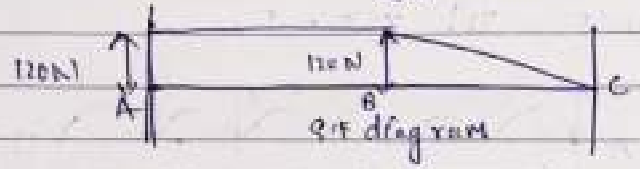
BM at A, $M_A = (5 \times 10) \times \left(\frac{10}{2}\right) = -250$ Nm

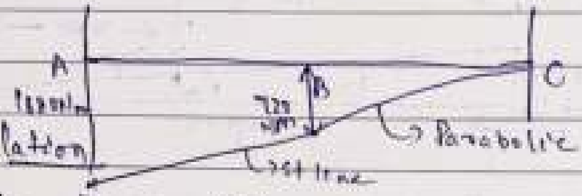


Date ___ / ___ / ___

Q) Draw the SF and BM diagram of the Cantilever Beam as shown below:-







S.F Calculation

S.F at C, F_C = 0N

S.F at B, F_B = + 120N

S.F at A, F_A = + 120N

B.M Calculation

B.M at C, $M_C = 0$

B.M at B, $M_B = (10 \times 12) \times \left(\frac{12}{2}\right)$

$= -720$ N/m

B.M at A, $M_A = (10 \times 12) \times (14)$

$= 120 \times 14$

$= -1680$ Nm

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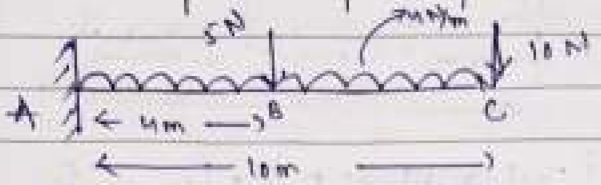
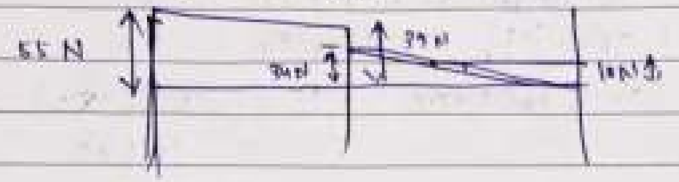
* Cantilever with point load and UDL :-





SF calculation :-

S.F at C (Just right of C), $F_C = 0N$

(Just left of C) $F_C = +10N$

S.F at B (Just right of B) $= 10 + (4\times6)$

$F_B = 10 + 24 = +34N$

(Just left of B) $= (10 + 24 + 5)N$

$F_B = +39N$

S.F at A, $F_A = 10 + 5 + 40$

$= +55N$

BM calculation :-

BM at C, $M_C = 0$

BM at B, $M_B = 10\times6 + \overset{12}{\cancel{24}} \times \frac{6}{\cancel{2}} = 60 + \overset{72}{\cancel{}} = 60 + 72$

$= -132 Nm$

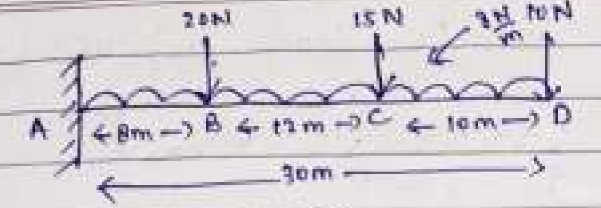
BM at A, $M_A = (10\times10) + (5\times4) + (\overset{20}{\cancel{40}}) \times \frac{10}{\cancel{2}}$

$= 100 + 20 + 200$

$= -320$

Page No.

Q)



S.F (Just right of D), $F_D = 0$

S.F (Just left of D), $F_D = 10N$

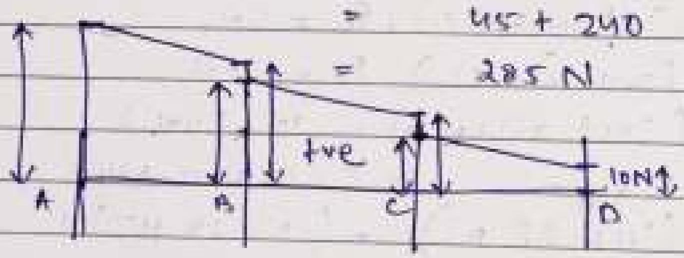
S.F (Just ~~left~~ right of C) $= F_C = 10 + (8 \times 10)$
$= 10 + 80$
$= 90 N$

S.F (Just left of C) $= F_C = 15 + 10 + (8 \times 10)$
$= 105 N$

S.F (Just right of B) $= F_B = 10 + 15 + (8 \times 22)$
$= F_B = 201 N$

S.F (Just left of B) $= F_B = 20 + 15 + 10 + (8 \times 22)$
$= 221 N$

S.F at A $= F_A = 20 + 15 + 10 + (8 \times 30)$
$= 45 + 240$
$= 285 N$



BM at D, $M_D = 0$

BM at C, $M_C = (10 \times 10) + (10 \times 8 \times \frac{10}{2})$
$= 100 + 400$
$= -500 Nm$

BM at B, $M_B = (15 \times 12) + (10 \times 22) + (22 \times 8 \times \frac{22}{2})$
$= -2336 Nm$

BM at A, $M_A = (20 \times 8) + (15 \times 20) + (10 \times 30) + (8 \times 30 \times \frac{30}{2})$
$= 160 + 300 + 300 + 3600$
$= -4360$

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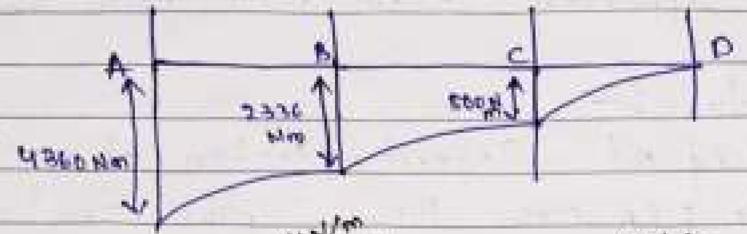
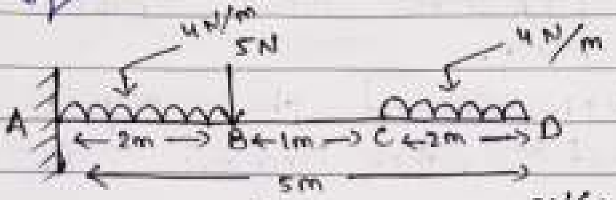
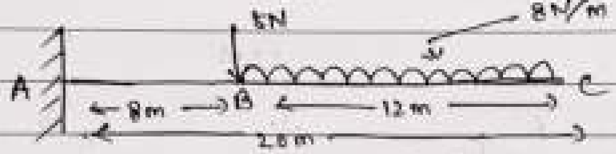

A
B
C
D
4360 Nm
2336 Nm
500 Nm

HW

Q1>


4 N/m
5N
4 N/m
A
← 2m → B ← 1m → C ← 2m → D
5m

Q2>


5N
8 N/m
A
← 8m → B ← 12m →
C
20m

Date ___/___/___

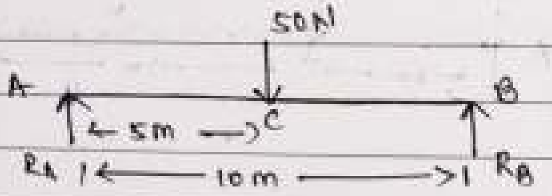
* Simply supported beam with point load:-

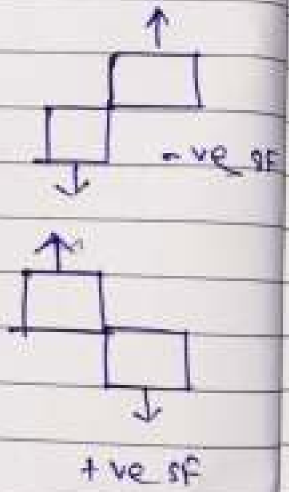
Procedure:-

a) To Find out Support reaction at A & B
Let R_A & R_B are the Support reaction at A & B respectively.

b) By taking moment about any one of the Support, then Find out R_A & R_B.

Q)





Taking moment about 'A',

$$+(50\times5)-(R_B\times10)=0$$
$$+250-(R_B\times10)=0$$
$$\therefore R_B=\frac{250}{10}=25\ N$$

We know that

$$R_A+R_B=50$$
$$R_A+25=50$$
$$R_A=25N$$

From SF diagram at point C S.F changes its Sign positive to negative, From this, we conclude that at point, Bending moment value will be maximum.

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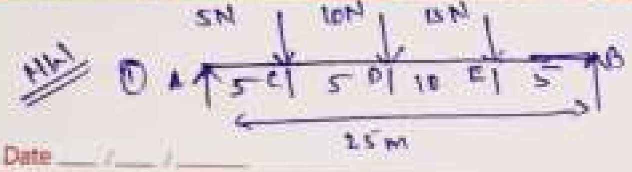
HW ①

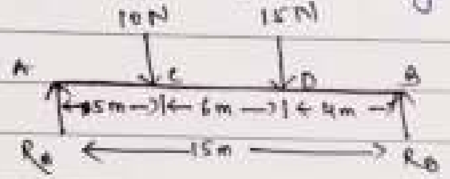


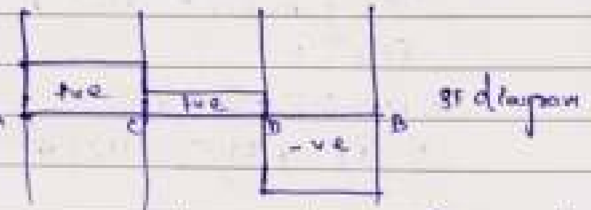
Date ___ / ___ / ___

SF Calculation :-

SF at A (Just left of A) , $F_A = 0$
SF at A (Just right of A) , $F_A = +25N$
SF at C (Just left of C) , $F_C = +25N$ 25 - 50 = ~~+25N~~
SF at C (Just right of C) , $F_C = 25 - 50 = -25N$
SF at B (Just left of B) , $F_B = -25N$ (25-50)
SF at B (Just right of B) , $F_B = 0$ (25-50+25)

Q)





Sol^n)

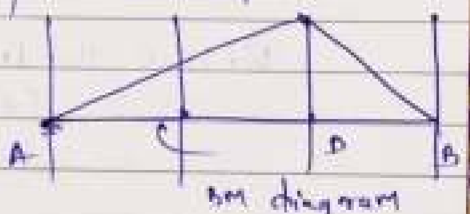
$\Sigma M_A = 0$

$\Rightarrow 10 \times 5 + 15 \times 11 - R_B \times 15 = 0$

$\Rightarrow 50 + 165 - R_B \times 15 = 0$

$\Rightarrow (R_B \times 15) = 215$

$\Rightarrow R_B = \frac{215}{15} = 14.33N$



We know that, $R_A + R_B = 25N$
$R_A + 14.33 = 25$
$R_A = 25 - 14.33$
$= 10.67N$

BM Calculation :-
BM at A, $M_A = 0$
BM at C, $M_C = 10.67 \times 5 = 53.35 Nm$
BM at D, $M_D = 10.67 \times 11 - 10 \times 6 = 57.37Nm$
BM at B, $M_B = (10.67 \times 15) - (10 \times 10) - (15 \times 4)$
$= 0.05 Nm$

SF Calculation :-
SF at A (Just left of A), $F_A = 0$
SF at A (Just right of A), $F_A = +10.67N$
SF at C (Just left of C), $F_C = +10.67N$
SF at C (Just right of C), $F_C = 10.67 - 10 = 0.67N$
SF at D (Just left of D), $F_D = 0.67N$
SF at D (Just right of D), $F_D = 10.67 - 10 - 15 = -14.33N$
SF at B (Just left of B), $F_B = -14.33N$
SF at B (Just right of B), $F_B = 10.67 - 10 - 15 + 14.33$
$= 0$

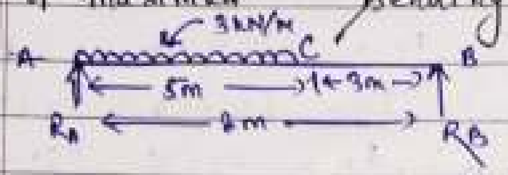
Page No.

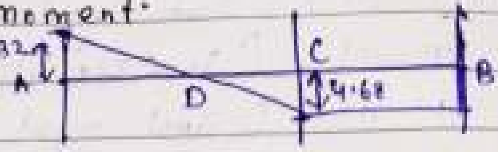
Saathi

Date ___/___/_____

Q> A Simply supported beam 8m long carrying a UDL of 3kN/m over a length of 5m from the left supported. Draw SF and BM diagram. Determine Position and value of maximum bending moment.

Sol>





Let R_A & R_B are the support reaction at A & B respectively.

Taking moment about A

$\Sigma M_A = 0$

$\Rightarrow 15 \times 2.5 - R_B \times 8 = 0$

$\Rightarrow R_B = \frac{37.5}{8} = 4.68\,N$

We know that $R_A + R_B = 15$

$R_A + 4.68 = 15$

$R_A = 15 - 4.68$

$= 10.32\,N$

SF Calculation:-

SF at A (Just left of A) $= F_A = 0$

SF at A (Just right of A) $= F_A = +10.32\,N$

SF at C, $F_C = 10.32 - 15 = -4.68\,N$

SF at B (Just left of B) $= F_B = 10.32 - 15 = -4.68\,N$

SF at B (Just right of B) $= F_B = 10.32 - 15 + 4.68 = 0$

From SF diagram at Point D, SF changes its sign from +ve to -ve, where BM value will be maximum.

To find out Point 'D', from similar Δ Principle

$\frac{AD}{10.32} = \frac{CD}{4.68}$ | $\frac{x}{10.32} = \frac{(5-x)}{4.68}$

Let $AD = x\,m$

$CD = (5-x)\,m$

$\Rightarrow 4.68x = 10.32 \times 5 - 10.32x$

$\Rightarrow 4.68x = 51.6 - 10.32x$

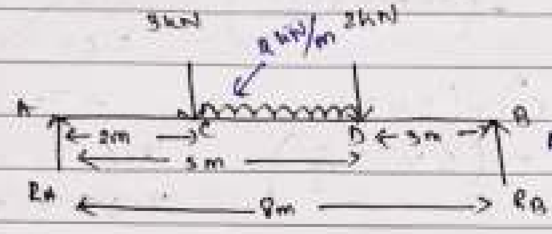
$\Rightarrow 15x = 51.6$

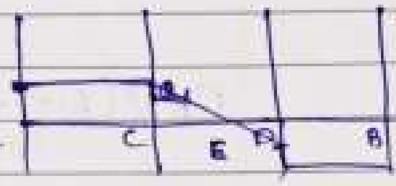
$\Rightarrow x = \frac{51.6}{15} = 3.44\,m$

from the shear force diagram, we get the Point E, where shear force changes its sign from +ve to −ve. There, value of BM will be maximum.

Date __/__/__

Q) A simply supported AB 8m long, carrying a point load 3kN at 2m from A and 3kN and UDL of 2kN/m. Determine the position and ~~value~~ magn of maximum Bending moment.





Sol→ Let R_A & R_B are support reaction.

Taking moment about A

$= (3\times2) + (2\times5) + (2\times3)\times3.5 - R_B\times8 = 0$

$= 6 + 10 + 21 - R_B\times8 = 0$

$= 37 - R_B\times8 = 0$

$= R_B\times8 = 37$

$= R_B = \frac{37}{8} = 4.62$ kN

We know $R_A + R_B = 3+2 + (2\times3)$

$R_A + 4.62 = 11$

$R_A = 11 - 4.62$

$= 6.38$ kN

S.F Calculation:-

SF at A (Just left of A) = 0

SF at A (Just right of A) = +6.38

SF at C (Just left of C) = +6.38

SF at C (Just right of C) = 6.38 − 3 = 3.38 kN

SF at D (Just left of D) = 6.38 − 3 − (2×3) = −2.62

SF at D (Just right of D) = 6.38 − 3 − (2×3) − 2 = −4.62

SF at B (Just left of B) = −4.62

SF at B (right) = 0kN

-ve +ve

Saathi

Date __ / __ / __

BM Calculation :-

$M_A = 0$

$M_C = +(6.38 \times 2) =$

$M_E = +(6.38 \times 3.69) - (3 \times 1.69) - \left(2 \times 1.69 \times \frac{1.69}{2}\right)$

$M_D = +(6.38 \times 5) - (3 \times 3) - \left(2 \times 3 \times \frac{3}{2}\right)$

$M_B = 0$

To find out CE

Let CE = x m

$\frac{CE}{3.38} = \frac{ED}{2.62}$

$\frac{x}{3.38} = \frac{3-x}{2.62}$

$\cancel{2.62 = 3.38 \times 3 - 3.38x}$

$\cancel{2.62 = 10.14 - 3.38x}$

$\cancel{-7.52 = -3.38x}$

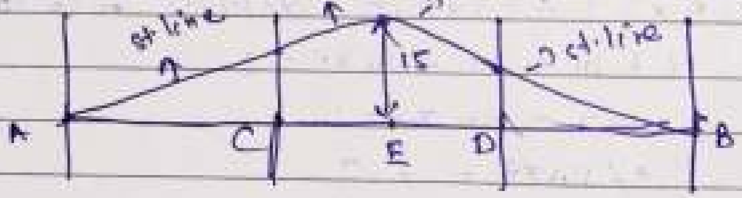
$2.62x = 3.38(3-x)$

$2.62x = 3.38 \times 3 - 3.38 \times x$

$2.62x = 10.14 - 3.38x$

$6x = 10.14$

$x = \frac{10.14}{6} = 1.69$ m



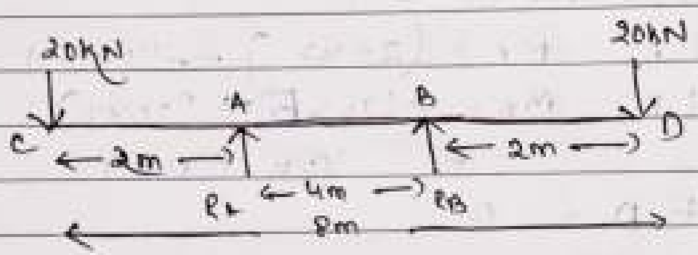
Q.1)

Sol)

Date __ / __ / ____

* SF & BM Diagram of over hanging beam:—

Q-1) An overhanging beam of length 8m is simply Supported over 4m apart and overhanged of 2m at each side. The beam Carries two Concentrated loads 20 kN at both End. Draw S.F and BM diagram



Sol) let R_A & R_B are Support reaction at A & B

Taking moment about A = $(20\times2) - (R_B\times4) + (20\times6) = 0$

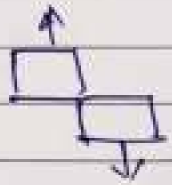
$= -40 - 4R_B + 120 = 0$

$= 40 - 4R_B = 0$

$= 80 = 4R_B$

$= \frac{80}{4} = R_B$

$= 20\ \text{kN} = R_B$

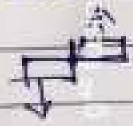


+ve SF

$R_A + R_B = 40$

$R_A + 20 = 40$

$R_A = \cancel{40+70 = 110\,kN}$ $40 - 20 = 20\,kN$



-ve SF

SF Calculation:—

SF at (left of C) = 0

(Right of C) = −20kN

SF at A (left of A) = −20kN

SF at A (right of A) = −20 + 20 = 0

SF at B (left of B) = 0

SF at B (right of B) =

SF at D (left of D) = −20 + 20 + 20 = 20kN

SF at D (right of D) = 0

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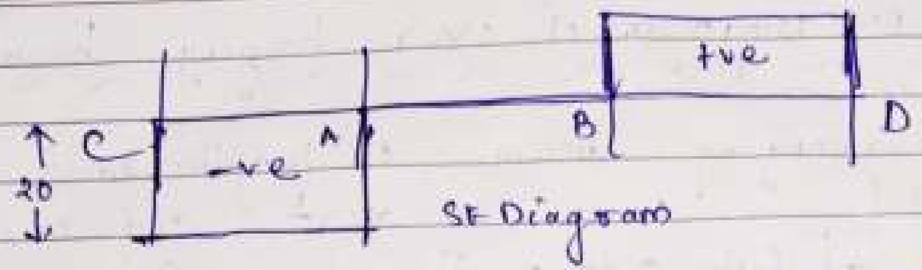
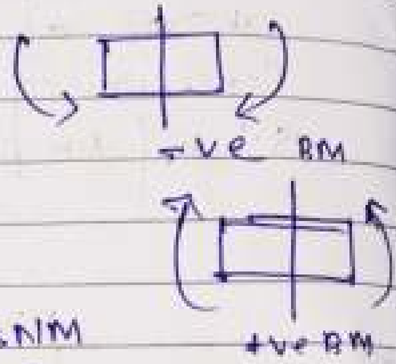


SF Diagram



BM Calculation:-

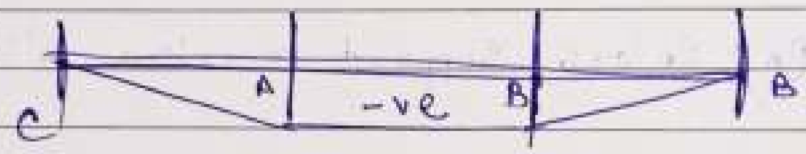
BM at C ; $M_C = 0$

BM at A, $M_A = (20 \times 2) = -40 kNM$

BM at B, $M_B = -(20 \times 6) + (20 \times 4)$

$= \cancel{40k}\ -40 kNM$

BM at D = 0



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Q-2) An over hanging beam loaded with three concentrated as shown in Figure. Draw SF and BM diagram and also locate point of Contraflexure.

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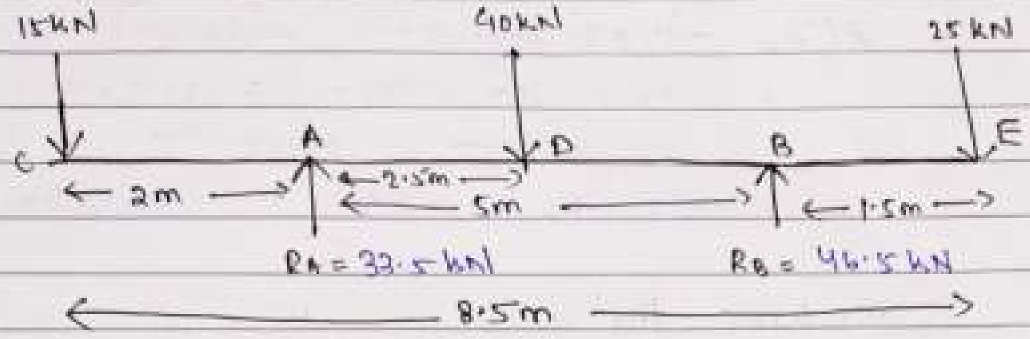

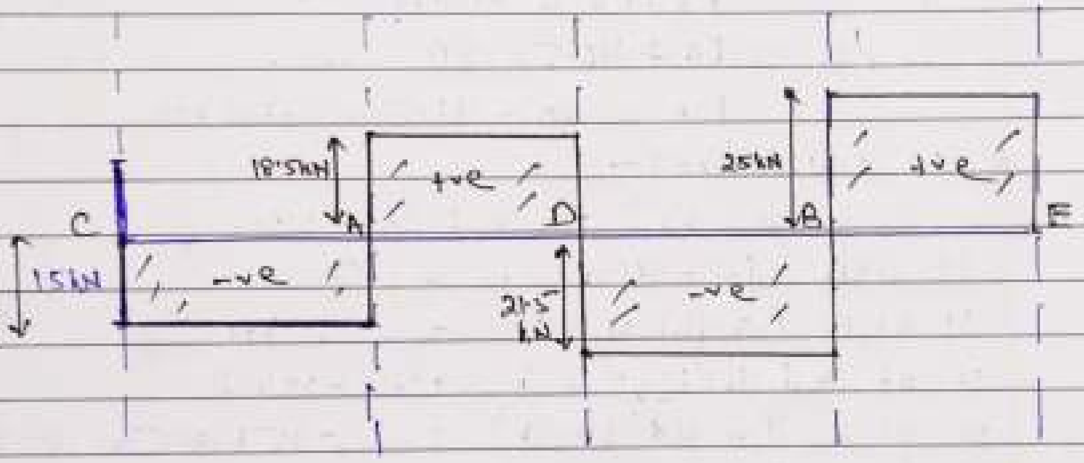

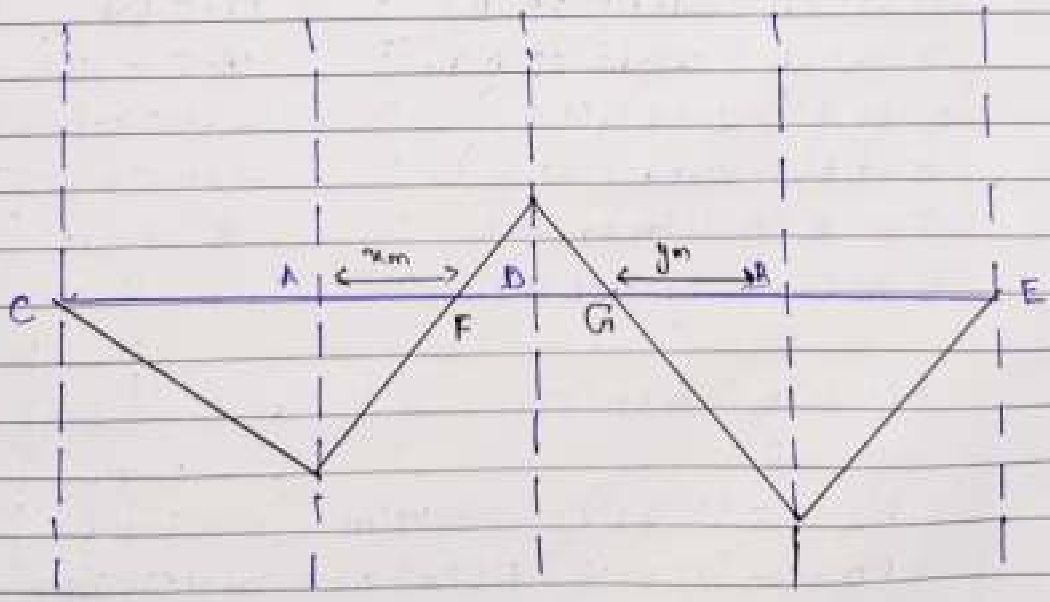

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Let R_A & R_B are the Support reaction at A & B respectively.

$$\Sigma M_A = -(15\times2)+(40\times2.5)-(R_B\times5)+(25\times6.5)=0$$

$$= -30+100-(R_B\times5)+162.5=0$$

$$= 232.5 = R_B\times5$$

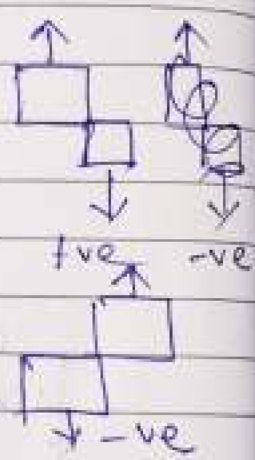
$$= \therefore R_B = \frac{232.5}{5} = 46.5\text{ kN}$$

We know that,

$$R_A+R_B = 80\text{ kN}$$

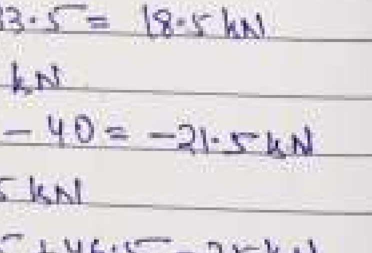
$$R_A+46.5 = 80$$

$$R_A = 80-46.5 = 33.5\text{ kN}$$



SF calculation:-

SF at C (left of C) = 0
SF at C (right of C) = −15 kN
SF at A (left of A) = −15 kN
SF at A (right of A) = −15+33.5 = 18.5 kN
SF at D (left of D) = 18.5 kN
SF at D (right of D) = 18.5 − 40 = −21.5 kN
SF at B (left of B) = −21.5 kN
SF at B (right of B) = −21.5+46.5 = 25 kN
SF at E (left of E) = 25 kN
SF at E (right of E) = 0



BM Calculation:-

$M_C = 0$

$M_A = -(15\times2) = -30\text{ kNm}$

$M_D = -(15\times4.5)+(33.5\times2.5) = 16.25\text{ kNm}$

$M_B = -25\times1.5 = -37.5\text{ kNm}$

$M_E = 0$

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Let the Point F is at distance of xm From A, where M_F is 0

$M_F = 0$

$\Rightarrow -\cancel{15x} - 15 \times (2+x) + 33.5x = 0$

$\Rightarrow -30 - 15x + 33.5x = 0$

$\Rightarrow -30 + 18.5x = 0$

$\Rightarrow 18.5x = 30$

$x = \frac{30}{18.5} = 1.62\,m$

Let the point E is at distance of ym From B, where MG is 0

MG = 0

$\Rightarrow -25 \times (1.5+y) + 46.5y = 0$

$\Rightarrow -37.5 - 25y + 46.5y = 0$

$\Rightarrow -37.5 + 21.5y = 0$

$\Rightarrow 21.5y = 37.5$

$y = \frac{37.5}{21.5} = 1.74\,m.$

From BM diagram we get two point of Contraflexure at point F & G

Point OF Contraflexure:-

It is a point, where BM Changes its sign from Positive to negative is Called Point of Contraflexure.

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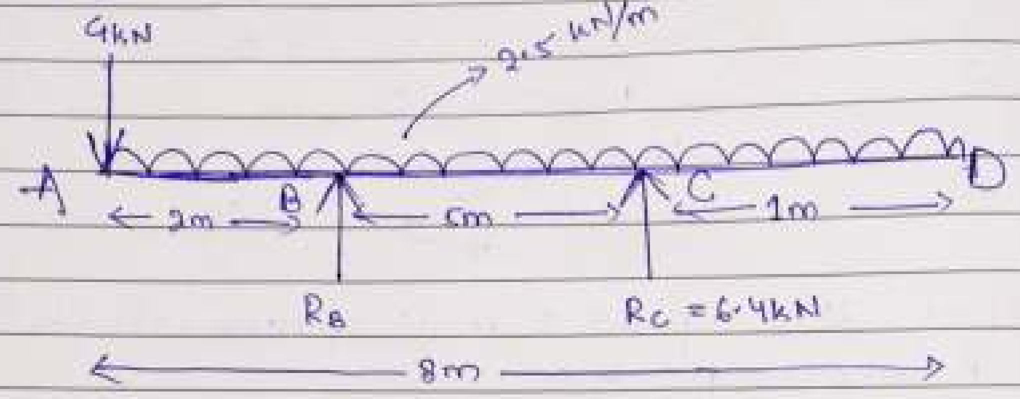
Q.27 Draw SF and BM diagram of an over hanging beam loading with UDL as shown in the fig. Find the point of contraflexure.

Sol:



Let R_B & R_C are the two support reaction at B & C.

$\Sigma M_B = 0$

$-(4\times2) + (2.5\times8)\times2 - R_C\times5 = 0$

$-8 + 40 - R_C\times5 = 0$

$32 = R_C\times5$

$\therefore\ R_C = \frac{32}{5} = 6.4$ kN

We know that

$R_B + R_C = 4 + (2.5\times8)$

$R_B + 6.4 = 24$

$R_B = 24 - 6.4$

$= 17.6$ kN

SF calculation :-

SF at A	$= 0$
SF at A	$= -4$ kN
SF at B	$= -9$ kN $\Rightarrow -4-(2.5\times2)$
SF at B	$= -4-5+17.6 = 8.6$ kN
SF at C	$= -4+17.6-(2.5\times7) = -3.9$ kN
SF at C	$= -3.9+6.4 = 2.5$ kN
SF at D	$= 0$

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Column & Strutt:—

Column and Strutt structural member, when it is placed other than vertical position, then it is called strutt

but when it is placed vertical position then it is called column.

There are two types of Column:—

a) Short Column
b) Long Column.

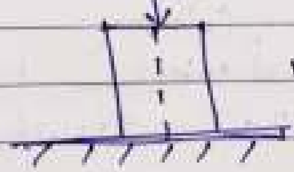
Generally, short column is Breakdown due to Crushing or Compressive failure.

Generally, Long Column is feld due to buckling.

Here, the topic is about long Column and the failure will be due to buckling.

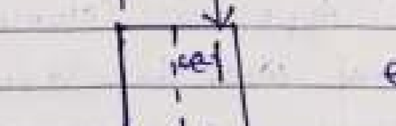
The load which is acting on a Column is purely vertical load on it is called axial load.

~~Ecc~~



w is axial load

Eccentric load:— when the load is acting away from the C.G then, is called Eccentric load



e → eccentricity.

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Buckling load or Critical load or Crippling load :-

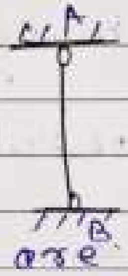
For a long column, the load is acting vertically downward direction, when load increases gradually, for a particular point, the column will start to buckle, corresponding to that load, when the buckling is started is known as buckling load.

Types of Column depending upon End Condition :-

There are 4 types of Column :-

(1) Both End hinged
(2) Both End fixed
(3) one End hinged and other End Fixed.
(4) one End Fixed other End Free

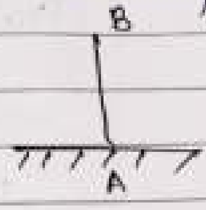
(1) It is a such type of Column, whose both Ends are hinged arrangement.



(2) It is such type of Column, whose both Ends are rigidly Fixed, with the Structure.

(3) It is a Such type of Column, in which, one End is rigidly Fixed with the Structure and other End having hinged arrangement

(4) It is a such type of column, whose one End is rigidly fixed with the structure and other End is Completely Free



During loading Conditions, when a Column is Subjected to a vertical load, the deflection of the Column will be different for different end Condition.

For, Calculation point of view

The original length of the Column => l

The Effective length or Equivalent length means Exactly amount of bending during a load applied Condition, is represented by (L)

l = original length

L = effective length

To get Effective length a Const. factor is multiplied with original length of the Column i.e. is =

$$\boxed{L = C \times l}$$

where c = Constant Factor

Value of 'c'	End condition
$C = 1$	both end hinged
$C = 1/2$	both End Fixed
$C = 1/\sqrt{2}$	one End hinged other End fixed
$C = 2$	one End fixed other End Free.

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Mathematically,

Critical load, $\boxed{P_{cr} = \frac{\pi^2 E I}{L^2}}$ → Euler's Formulae

where, E = young's modulus
L = Effective length of the column.
(M.I) I = least moment of Inertia

2nd Condition

$L = C \times l$

$L = \frac{1}{2} \times l$

$L = \frac{l}{2}$, $P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 E I}{\left(\frac{l}{2}\right)^2} = \frac{4\pi^2 E I}{l^2}$

3rd Condition

$L = C \times l$

$L = \frac{1}{\sqrt{2}} \times l = \frac{l}{\sqrt{2}}$, $P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 E I}{\left(\frac{l}{\sqrt{2}}\right)^2} = \frac{2\pi^2 E I}{l^2}$

4th Condition

$L = C \times l$

$L = 2l$, $P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 E I}{4l^2}$

1st Condition

$L = C \times l$

$L = 1 \times l = l$

$P_{cr} = \frac{\pi^2 E I}{L^2} = \frac{\pi^2 E I}{l^2}$

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Q-1) A Steel Column is of length 8m and dia 600mm with both Ends hinged. Determine Crippling load by using, Euler's Formulae

Take, $E = 2 \times 10^5 \ N/mm^2$

Given,—

Sol) $l = 8m$

$d = 600mm$

$E = 2 \times 10^5 \ N/mm^2$

$I = \cancel{\frac{\frac{\pi}{4}(d)^2}{64}} = \cancel{4417.86}$ $\quad \frac{\pi d^4}{64} = \frac{\pi (600)^4}{64}$

$= 6361725124 \ mm^4$ ~~17671.45~~

$P_{cr} = \frac{\pi^2 E I}{l^2}$

$= \frac{\pi^2 \times 2\times10^5 \times 6361725124}{(8\times1000)^2}$ $\quad \left(\frac{\frac{N}{mm^2} \times mm^4}{mm^2} \right)$

$= 196211594.6 \ N$ ~~10^3~~

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Date __/__/____ Bending Stress

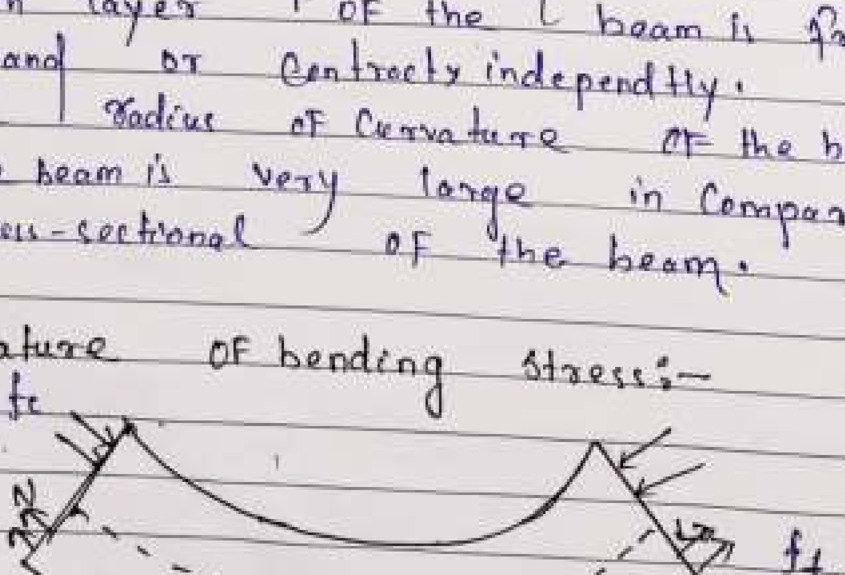
Bending Stress is the stress, which is the resistance occur by the internal stresses to the bending is known as bending stress.

The resistance offered by the internal stress to the shear force is known as shear stress.

Imp Question — Assumption to the theory of Simple Bending:-

1) The material of the beam is homogeneous and isotropic.
2) The transverse Section of the beam before bending is a plane and also plane after bending.
3) The value of young's modulus is same in tension and Compression.
4) The material of the beam obeys hooke's law and it is stressed within elastic limit.
5) Each layer of the beam is free to expand or contracts independtly.
6) The radius of Curvature of the beam of the beam is very large in Comparing to the Cross-sectional of the beam.

Nature of bending stress:-



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The nature of bending stress as shown in the figure i.e is from neutral line to top fibre is Compressive (f_c)

The nature of stress from neutral line to top fiber is Tensile (f_t)

Imp

Sure

Bending Equation:-

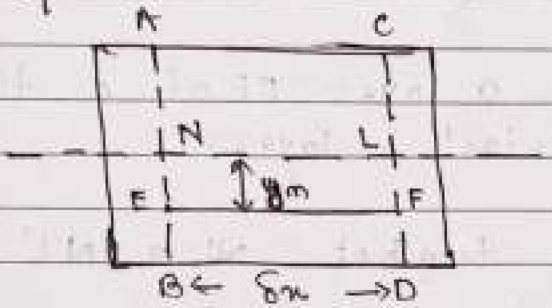

Fig-1 (Before bending)

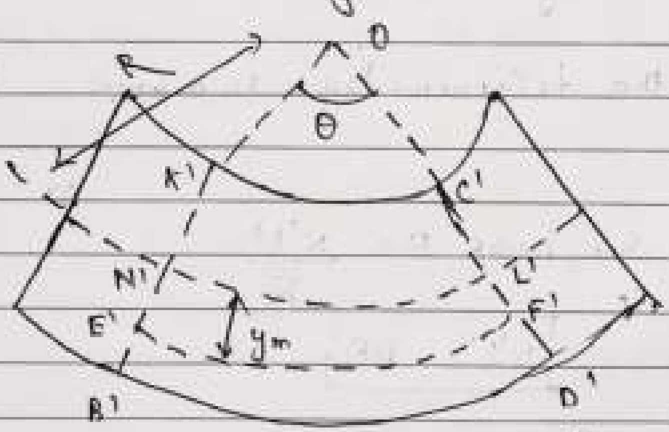

Fig-2 (After bending)

Let us consider a beam before bending as shown in Fig-1 and after bending in Fig-2

Let δx a small length bet^n 2 transverse Section AB & CD.

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Let after bending, the layer remains on the plane A'B' and C'D'

Let the planes A'B' and C'D' meet at point O making an angle θ

Let the radius of the neutral layer N'L' is Equal to R.

Consider, a layer EF at a distance of y from neutral layer.

Original length of $NL = N'L' = \delta x$

From the Fig - (i), N'L'

from the trigonometry we know - $\theta = \frac{l}{R}$

or $l = R\theta$

So, here $\theta = \frac{N'L'}{R}$

$$\boxed{N'L' = R\theta}$$

original length of $NL = \delta x$
Change in length $N'L' = R\theta$

but, we know neutral line is ineffective.

so, $\boxed{NL = \delta x = N'L' = R\theta}$

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From Fig-1 $EF = \delta x = R\theta$

After bending, the layer becomes, $E'F'$
Hence, the change in length of layer EF
$EF = E'F' - EF$

Hence, $E'F'$

From the trigonometry, we know: $\theta = \frac{L}{R}$

or $L = R\theta$

so, here $\theta = \frac{E'F'}{R+y}$

$\therefore E'F' = (R+y)\theta$

we know, change in length of the layer EF=

$EF = E'F' - EF$
$= (R+y)\theta - R\theta$
$= \cancel{R\theta} + y\theta - \cancel{R\theta}$
$= y\theta$

$\boxed{\text{Change in length of EF} = y\theta}$

So, strain on the layer EF $= \frac{\text{Change in length of EF}}{\text{original in length of EF}}$
(e)

$e = \frac{y\theta}{R\theta}$

$\boxed{e = \frac{y}{R}}$

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$$\boxed{\frac{f}{E} = \frac{y}{R}} \quad \text{or} \quad \boxed{\frac{f}{y} = \frac{E}{R}}$$

, where f = bending Stress
E = young's modulus
y = distance of top Fiber From NL
or
distance of bottom Fiber From NL.
R = radius of Curvature

$$f = \frac{E}{R} \times y$$

E = Constant
R = Constant

$$f = y \times c$$

$\boxed{f \propto y}$ ⇒ when y minimum, stress is also minimum
→ when y maximum, stress is also maximum

i.e means stress ∝ Constant

So, at the Extreme top layer stress value is maximum and at the extreme bottom layer, Stress is also maximum & at the neutral line stress is zero.

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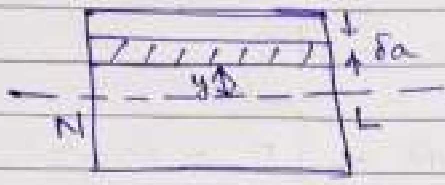
* 2nd part of bending Equation :—



Let us Consider a rectangular beam Cross section, as shown in the Figure, NL is the neutral line.

Consider, an Elementary strip at a distance of y from neutral line.

Area of the Elementary strip = δa $f = \frac{P}{A}$

The Force on this strip = $f \times \delta a$ —— 1 $[P = f \times A]$

Also, we know

$$\frac{f}{y} = \frac{E}{R}$$

$$f = \frac{E}{R} \times y \quad \text{—— (2)}$$

Now, Putting the value of f on Eqn (1) we get

∴ Force on the elementary strip = $\frac{E}{R} \times y \times \delta a$

Bending Moment of this Force about neutral axis = $\left(\frac{E}{R} \times y \times \delta a\right) \times y$

B.M. of this Elementary Strip about NL = $\frac{E}{R} \times y \times \delta a \times y$

$$\boxed{\delta m = \frac{E}{R} \times \delta a y^2}$$

It is representive for Elementary strip

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Date __/__/__ (BM)

So, to find out moment of resistance, for the complete beam section by integrating both sides of the Equation

$$\int \delta m = \int \frac{E}{R} \times \delta a y^2$$

$$M = \frac{E}{R} \int \delta a y^2$$

We know that
[∵ From the Moment of Inertia i.e $I = \int \delta a y^2$]

$$M = \frac{E}{R} I$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{f}{y}}$$

Where, M = Max Bending moment of the beam
I = moment of inertia of the beam section
E = Young's modulus
R = Radius of curvature
f = bending stress
y = distance of top fiber from NL (Neutral line)

* **Moment of resistance or maximum bending moment**

Moment of resistance of a beam is a fixed value, it is depending upon the structure of the beam on, which, it is prepared, but loading condition on the beam is different and it is having no limitation. For a particular value of the load, the BM develop is a fixed value, but resistance of a beam, upto which, the beam will sustain i.e not to fell

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So, For numerical point of view, solving the problem max bending moment value is - Equate with moment of resistance.

* Maximum bending moment value for Cantilever beam

max bending moment $= W \times l = M.R$

(Point load)

* Maximum bending moment value for simple supported beam

max bending moment $= \frac{WL}{4}$

(UDL)

* maximum bending moment value for simple supported beam

$$\text{maximum bending moment} = \frac{wl}{2} \times \frac{l}{2} - \frac{wl^2}{8}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$= \frac{2wl^2 - wl^2}{8} = \frac{wl^2}{8}$$

* Value of M

1) For Cantilever beam with UDL — $\frac{WL^2}{2}$

1) For cantilever — $W \times l$

2) For simply supported beam (Point load) $= \frac{WL}{4}$

3) For simply supported beam (UDL) $= \frac{WL^2}{8}$

4)

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Section Modulus :- (Z)

It is defined as the ratio of moment of inertia to the distance of top layer or bottom layer from neutral line.

Mathematically, $\boxed{Z = \frac{I}{y}}$

It's only for bending condition of beam.

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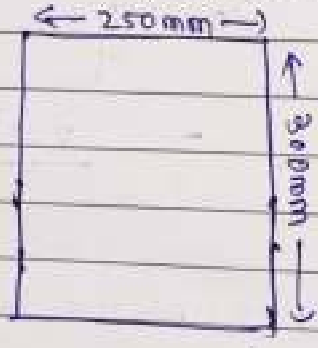
Numerical :-

Q-1) A beam 250 mm wide and 300 mm deep is used to carry a uniformly distributed load of 1000 N/m over a span of 5m. Find out the max stress developed in the beam?

Sol)



Span = Simply supported beam

(Load) $W = 1000$ N/m

$L = 5$m

∴ Bending stress $= f$?

$M = \frac{wL^2}{8}$, $y = \frac{300}{2} = 150$ mm

$I = \frac{bd^3}{12}$

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$\therefore \frac{M}{I} = \frac{f}{y}$

$M = \frac{wl^2}{8} = \frac{1000 \times 25}{8} = 3125 \text{ N-m}$

$= 3125000 \text{ N-mm}$

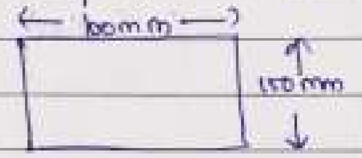
$I = \frac{bd^3}{12} = \frac{250 \times 300^3}{12} = 562500000 \text{ mm}^4$

$y = 150 \text{mm} \therefore f = \frac{3125000}{526500000} \times 150 \quad \frac{\text{N-mm}}{\text{mm}^4} \times \text{mm}$

$= 0.89 \text{ N/mm}^2$

Q) If a beam of Section 100 mm wide & 150mm depth carries is a load of 5000N/m & the bending stress is not to excde. 110 N/m² . Find the span of beam.

Given



Sol) $b = 100 \text{mm}$, $y = \frac{150}{2} = 75 \text{ mm}$

$d = 150 \text{ mm}$

$w = 5000 \text{ N/m}$

$f = 110 \text{ N/m}^2$

$\therefore$ Moment of inertia of the beam is

$\frac{\text{N-mm}}{\text{mm}^4} \times \text{mm}$

$I = \frac{bd^3}{12} = \frac{100 \times (150)^3}{12} = 28125000 \text{ mm}^4$

We know that

$\left(\frac{\text{N/mm}^2}{\text{mm}} \times \text{mm}^4 \right)$

$\frac{f}{y} = \frac{M}{I}$

$\therefore M = \frac{f}{y} \times I = \frac{110}{75} \times 28125000 = 41250000 \text{ N-mm}$

We know that

$M = \frac{wl^2}{8} = \cancel{5000} \times$

$41250000 = \frac{5000}{8} \times l^2$

1406×10^3

$14062500 \therefore l^2 = \frac{41250000 \times 8}{5000}$

$l^2 = 66000$

$l = \sqrt{66000}$

$= 20\sqrt{165}$

$\approx 256.90 \text{ m}$

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Combined axial & bending stress of column

When a structural member is placed vertically & load is acting Exactly at it's CG is called ~~stress~~ direct stress.

but when load is acting out of C.G., then the stress developed are both direct & bending stress.

$$F_D = \frac{P}{A} \quad \boxed{A = b \times d} \quad — (1)$$

when the load is acting out of C.G, then it is called Eccentric load.

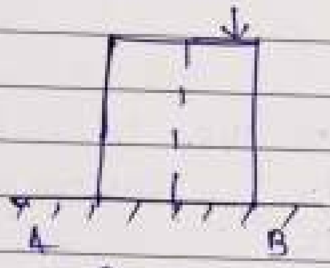

Fig-1

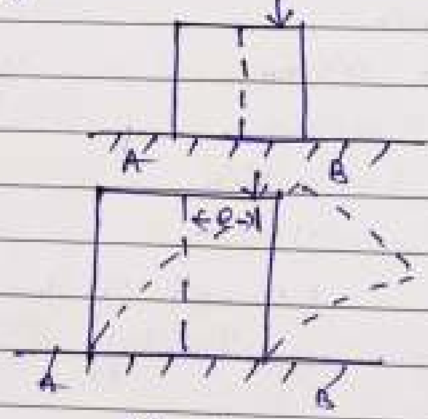

Fig-2

In, this condition, their are 2 types of stress, will be developed i.e is direct stress as well as bending stress

$$\boxed{F_d \, \& \, F_b} \quad — (2)$$

According to the diagram at the End B, intensity of stress is maximum and at the End A, intensity of stress is minimum.

So, methematically,

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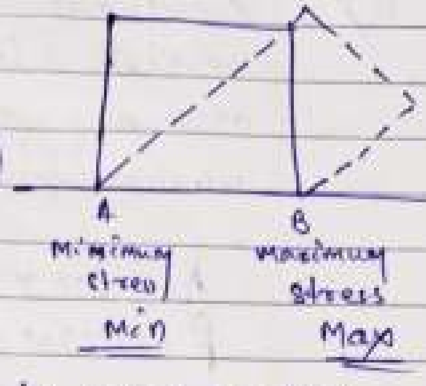
$F_{max} = F_d + F_b$

$F_{min} = F_d - F_b$



Nature of F_{max} is Compressive (+ve)

Nature of F_{min} is tensile (-ve)

The bending stress F_b Can be Calculated by using bending Equation.

$$\frac{F}{y} = \frac{M}{I}$$

$$\therefore F = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

$$\therefore \boxed{F_b = \frac{M}{Z}} \quad , \frac{I}{y} = Z = \text{Section modulus of beam Cross section / Column Section.}$$

Where, I = Moment of Inertia of Cross-section

y = distance of top layer from NL

$$F_b = \frac{M}{Z}$$

$$F_{max} = \frac{P}{A} + \frac{M}{Z}$$

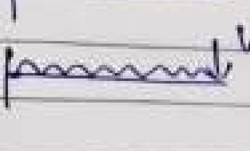
$$F_{min} = \frac{P}{A} - \frac{M}{Z}$$, where M is bending moment

For M = bending moment

$= P \times e$

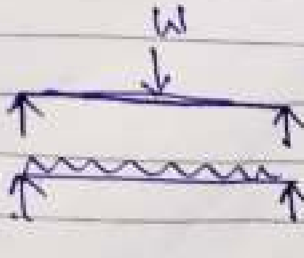
Where, P = load

e = Eccentricity.



$M_{max} = W \times L$

W kN/m $\quad M_{max} = \frac{WL^2}{2}$



$M_{max} = \frac{WL}{4}$

$M_{max} = \frac{WL^2}{8}$

Saathi

Date __ / __ / ____

Q/ A Square Column 250mm X 250mm Carries a vertical load of 120kN at a distance of 60cm from axis as shown in Fig. Find the maximum & minimum $\frac{\text{bending}}{\wedge}$ Stress induced in the Section.

Sol) $A = 250 \times 250$

$P = 120\,kN$

$I = \frac{bd^3}{12} = \frac{250 \times (250)^3}{12}$

$= 325520833.3$

$F_a = \frac{P}{A} = \frac{120}{250 \times 250} = \frac{120}{6250} = 1.92 \times 10^{-3}$

$F_b = \frac{M}{Z} = \frac{P \times e}{I/y} = \frac{120 \times 60}{I/125} = 2.7648 \times 10^{-3}$

$F_{max} = F_a + F_b$

$= 1.92 \times 10^{-3} + 2.7648 \times 10^{-3}$

$= 4.6848 \times 10^{-3}$

$F_{min} = F_a - F_b$

$= -8.448 \times 10^{-4}$

Date ___/___/___

8) A Solid rectangular Column 20cm wide & 15cm thick is Carrying a vertical load of 10kN. At Eccentricity of 5cm in a plane bisecting the thickness. Determine the max^n and min^m intensity of stress.

Stresses on thin cylindrical shell

Date __/__/__

Torsion

$$\boxed{t = \frac{D}{20}}$$

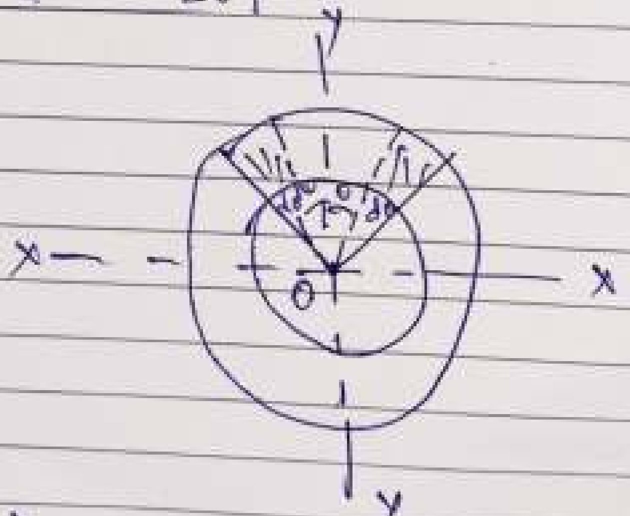
It is called thin cylinder because thickness is $\frac{1}{20}$th of diameter.

· There are two types of stresses developed :-

① Hoop Stress or Circumferential stress
② Longitudinal stress

* **Determination of Hoop Stress**

$$\boxed{\sigma_y = \frac{PD}{2t}} \longrightarrow \text{Hoop Stress}$$



Let us consider the section x-x divide the shell in two parts at any angle θ

In any angle θ, at either side of y-y consider two elementary strip which one making angle dθ at O / Center O.

Saathi

Date __/__/__

Let P = Fluid pressure

Let dF = Elementary normal force acting on this Elementary strip

We know, Force = Pressure × Area

$dF = \text{Pressure} \times \text{Elementary area}$

$dF = P \times (r\,d\theta \times l)$

The Elementary resultant force along y-y =

$= dF\cos\theta + dF\cos\theta$

$= 2dF\cos\theta \longleftarrow (i)$

Put the value of dF on eqn (i), we get

$= 2P \times r \times d\theta \cdot l \cdot \cos\theta$

Now, total value of the force along y-axis for bursting the cylinder

by integrating

$\Rightarrow \int dF = \int_0^{\pi/2} 2 \times P \times r \cdot d\theta \times l \times \cos\theta$

$\Rightarrow F = 2Prl \int_0^{\pi/2} \cos\theta \, d\theta$

$F = 2Prl \, [\sin\theta]_0^{\pi/2}$

$F = 2Prl$

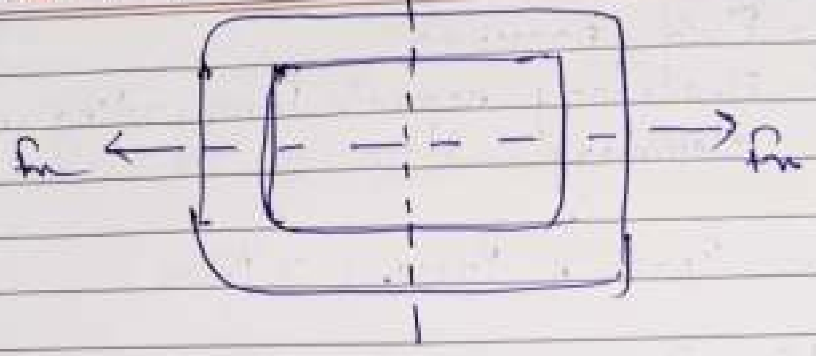
$\boxed{f_y = Pdl}$

The area which is responsible for ~~bursting~~ bursting is 2tl

So, Hoop stress $\boxed{f_y = \frac{\text{Force}}{\text{Area}} = \frac{Pdl}{2lt} = \frac{PD}{2t}}$

Longitudinal stress

Date __/__/__



Let us consider a section y-y where the force fn is acting along its length, and the stress, which is develop, is known as longitudinal stress.

Determination of longitudinal stress

Let d is diameter of shell
P is fluid pressure
t is thickness

$\therefore$ Tensile force f_n = Pressure $\times$ Area

$= P \times \frac{\pi}{4}(d)^2$

The area which is responsible for bursting is $= \pi d \times t$

So, longitudinal stress $= f_n = \frac{\text{Force}}{\text{Area}}$

$= \frac{P \times \frac{\pi}{4}(d)^2}{\pi d t}$

generally takes place due to hoop stress

$$\boxed{f_n = \frac{Pd}{4t}}$$

Date ___/___/___

Saathi

Q) Pressure inside a thin cylinder is 2115 Pa and its dia is 1m, if thickness of the cylinder wall is 5mm Determine hoop stress and longitudinal stress.

Sol) $P = 2115\,Pa = 2115\ N/m^2$

$D = 1m$

$t = 5mm = \frac{5}{1000} = 0.005m$

$$f_y = \frac{PD}{2t} = \frac{2115 \times 1}{2 \times 0.005} = 211500\ N/m^2$$

$$f_x = \frac{PD}{4t} = \frac{2115 \times 1}{4 \times 0.005} = 105750\ N/m^2$$

Q) A steel cylinder contains some fluid pressure and its dia is 1.5 m. If, the thickness of cylinder wall is 4mm. Determine the safe pressure.

Assume, maximum allowable tensile stress is $80\ N/mm^2$

$f_y = f_x = 80\ N/mm^2$

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